Practice with Probability Distributions

Your group has a set of cards. Assign one person the role of “dealer” and one the role of “recorder.”

How many cards are in your set? _______

How many black cards are in your set? _______

If you draw one card at random, what is the probability of drawing a black card? _______

**Step 1: Data Gathering**

**Dealer:**
1. Shuffle the deck.
2. Deal out 5 cards.
3. Count the number of black cards.

**Recorder:**
Put a tally next to the #of black cards.

**CLASS DATA:**
Repeat 10 times. Then switch roles and repeat 10 times. Your frequencies should add up to 20.

**Step 2: Class Totals**

Recall that:

\[ n = \sum f \quad P(x) = \frac{f}{n} \]

FIND the mean of the probability distribution (the expected value).

\[ \mu = \sum xP(x) \]

**DOES THIS MAKE SENSE?**

Usually the mean comes up to about 3. Most students are able to say “Well if 60% of the cards are black, it makes sense that 3 out of 5 are black.”

FIND the standard deviation of the probability distribution (the risk).

\[ \sigma = \sqrt{\sum x^2 P(x) - \mu^2} \]
Step 3: Does this experiment satisfy the conditions of a binomial experiment? If not, how could we CHANGE it so that it would satisfy the conditions?

*THIS IS IMPORTANT*  NO. This is NOT a binomial experiment, because the probability of success is not the same for each trial. We are dealing out five cards without replacement for our five trials. So, the probability of success is changing each time instead of remaining constant. To fix this, we should shuffle the deck, deal a card, note if black/red, and REPLACE CARD. We would need to do this 5 times for 5 trials.

\[
n = \square \\
x = \text{the \# of black cards} \\
p = \text{probability of black card} = \square \\
q = \square
\]

Could we use the binomial distribution table to calculate the probabilities in this situation or do we need to use the formula?

\[P(x) = C_{n,x} \cdot p^x q^{n-x}\]

Step 4: Find the mean of this probability distribution.

Expected Value/Mean

\[\mu = np\]

Standard Deviation

\[\sigma = \sqrt{npq}\]

Step 5: Make a histogram for the probability distribution.

Step 6: Be the teacher. Make up some questions.

a) \(P(\square) = \square\)

b) \(P(\square \text{ or } \square) = \square\)

c) \(P(\square \text{ to } \square) = \square\)

d) \(P(\text{at least } \square) = \square\)

e) \(P(\text{no more than } \square) = \square\)
Example 2:

**Step 1** Answer the question:

If you play “Rock, Paper, Scissors”, what is the probability of winning?

Rock defeats Scissors
Scissors defeats Paper
Paper defeats Rock

Think about it:

If YOU PLAY and THEY PLAY then you WIN/LOSE number of “wins” number of “possibilities”

\[
P(\text{win}) = \frac{\text{number of "wins"}}{\text{number of "possibilities"}}
\]

Great place to have a discussion about ties. Most students will say that the complement of “Win” is “Lose.” The complement of “Win” is “Not Win” so that would include “Lose” (example You: Rock/Them: Paper) and “Tie” (example You: Rock/Them: Rock)

**Step 2** In a round of 12 “rock paper scissors” matches, how many do you expect to win? What is the standard deviation?

**Step 3** Play 12 rounds of rock paper scissors: How many did you win? ______

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<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<th>8</th>
<th>9</th>
<th>10</th>
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<th>12</th>
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</thead>
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Put a W or X to keep track of “WIN” or “NOT WIN”

**Step 4** What is the probability of winning ______ out of 12 rounds of rock paper scissors? ______

\[
P(x) = C_{n,x} \cdot p^x q^{n-x}
\]

I have them calculate the probability of winning \(X\) = their value from step 3.

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Step 5: If you play rock-paper-scissors, your probability of winning is: _______
Fill in the binomial distribution table to represent the probability of winning “x” times out of 12 rounds.

<table>
<thead>
<tr>
<th>#wins X</th>
<th>P(x)</th>
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<tbody>
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<td>0</td>
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</table>

Graph the distribution. Why do you think it looks this way?

Get answers from step 4 to fill in as many values as possible. Assign students X values to finish filling out the table.

Step 6: Be the teacher. Make up some questions.

a) P( _____ ) = _______

b) P( _____ or _____ ) = _______

c) P( _____ to _____ ) = _______

d) P(at least _____ ) = _______

e) P(no more than _____ ) = _____