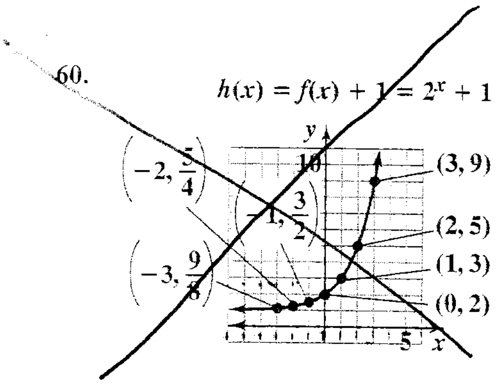
60.



Chapter 3 Review Exercises

1. $f(x) = -(x+1)^2 + 4$

vertex: (–1, 4)

x-intercepts:

$0 = -(x+1)^2 + 4$

$(x+1)^2 = 4$

$x + 1 = \pm 2$

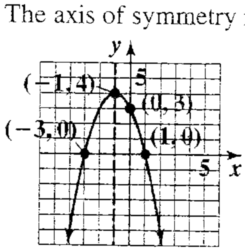
$x = -1 \pm 2$

$x = -3$ or $x = 1$

y-intercept:

$f(0) = -(0+1)^2 + 4 = 3$

The axis of symmetry is $x = -1$.



$f(x) = -(x+1)^2 + 4$

domain: $(-\infty, \infty)$ range: $(-\infty, 4]$

2. $f(x) = (x+4)^2 - 2$

vertex: (–4, –2)

x-intercepts:

$0 = (x+4)^2 - 2$

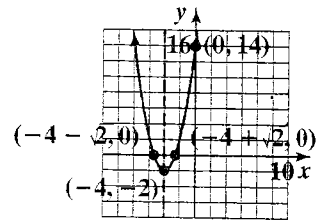
$(x+4)^2 = 2$

$x + 4 = \pm\sqrt{2}$

$x = -4 \pm \sqrt{2}$

y-intercept:

$f(0) = (0+4)^2 - 2 = 14 = -1$



$f(x) = (x+4)^2 - 2$

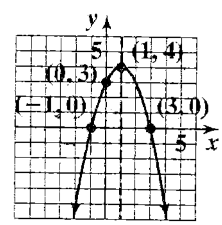
The axis of symmetry is $x = -4$.

domain: $(-\infty, \infty)$ range: $[-2, \infty)$

3. $f(x) = -x^2 + 2x + 3$

$= -(x^2 - 2x + 1) + 3 + 1$

$f(x) = -(x-1)^2 + 4$



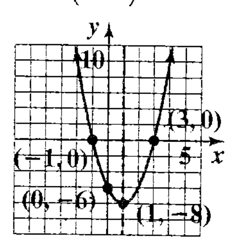
$f(x) = -x^2 + 2x + 3$

domain: $(-\infty, \infty)$ range: $(-\infty, 4]$

4. $f(x) = 2x^2 - 4x - 6$

$f(x) = 2(x^2 - 2x + 1) - 6 - 2$

$2(x-1)^2 - 8$



$f(x) = 2x^2 - 4x - 6$

axis of symmetry: $x = 1$

domain: $(-\infty, \infty)$ range: $[-8, \infty)$

5. $f(x) = -x^2 + 14x - 106$

a. Since $a < 0$ the parabola opens down with the maximum value occurring at

$$x = -\frac{b}{2a} = -\frac{14}{2(-1)} = 7.$$

The maximum value is $f(7)$.

$f(7) = -(7)^2 + 14(7) - 106 = -57$

b. domain: $(-\infty, \infty)$ range: $(-\infty, -57]$

6. $f(x) = 2x^2 + 12x + 703$

a. Since $a > 0$ the parabola opens up with the minimum value occurring at

$$x = -\frac{b}{2a} = -\frac{12}{2(2)} = -3.$$

The minimum value is $f(-3)$.

$$f(-3) = 2(-3)^2 + 12(-3) + 703 = 685$$

b. domain: $(-\infty, \infty)$ range: $[685, \infty)$

7. a. The maximum height will occur at the vertex.

$$f(x) = -0.025x^2 + x + 6$$

$$x = -\frac{b}{2a} = -\frac{1}{2(-0.025)} = 20$$

$$f(20) = -0.025(20)^2 + (20) + 6 = 16$$

The maximum height of 16 feet occurs when the ball is 20 yards downfield.

b. $f(x) = -0.025x^2 + x + 6$

$$f(0) = -0.025(0)^2 + (0) + 6 = 6$$

The ball was tossed at a height of 6 feet.

c. The ball is at a height of 0 when it hits the ground.

$$f(x) = -0.025x^2 + x + 6$$

$$0 = -0.025x^2 + x + 6$$

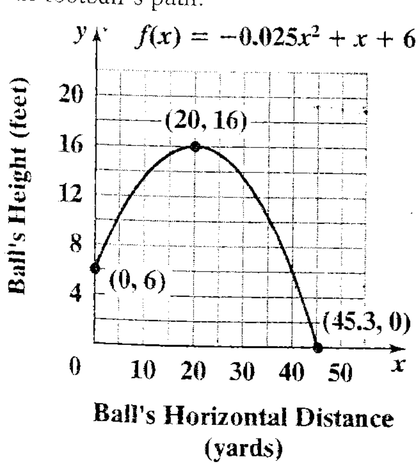
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(-0.025)(6)}}{2(-0.025)}$$

$x \approx 45.3, -5.3$ (reject)

The ball will hit the ground 45.3 yards downfield.

d. The football's path:



8. Maximize the area using $A = lw$.

$$A(x) = x(1000 - 2x)$$

$$A(x) = -2x^2 + 1000x$$

Since $a = -2$ is negative, we know the function opens downward and has a maximum at

$$x = -\frac{b}{2a} = -\frac{1000}{2(-2)} = -\frac{1000}{-4} = 250.$$

The maximum area is achieved when the width is 250 yards. The maximum area is

$$A(250) = 250(1000 - 2(250))$$
$$= 250(1000 - 500)$$
$$= 250(500) = 125{,}000.$$

The area is maximized at 125,000 square yards when the width is 250 yards and the length is $1000 - 2 \cdot 250 = 500$ yards.

9. Let x = one of the numbers
Let $14 + x$ = the other number

We need to minimize the function

$$P(x) = x(14 + x)$$
$$= 14x + x^2$$
$$= x^2 + 14x.$$

The minimum is at

$$x = -\frac{b}{2a} = -\frac{14}{2(1)} = -\frac{14}{2} = -7.$$

The other number is $14 + x = 14 + (-7) = 7$.

The numbers which minimize the product are 7 and -7. The minimum product is $-7 \cdot 7 = -49$.

10. $f(x) = -x^3 + 12x^2 - x$

The graph rises to the left and falls to the right and goes through the origin, so graph (c) is the best match.

11. $g(x) = x^6 - 6x^4 + 9x^2$

The graph rises to the left and rises to the right, so graph (b) is the best match.

12. $h(x) = x^5 - 5x^3 + 4x$

The graph falls to the left and rises to the right and crosses the y-axis at zero, so graph (a) is the best match.

13. $f(x) = -x^4 + 1$

$f(x)$ falls to the left and to the right so graph (d) is the best match.

14. The leading coefficient is –0.87 and the degree is 3. This means that the graph will fall to the right. This function is not useful in modeling the number of thefts over an extended period of time. The model predicts that eventually, the number of thefts would be negative. This is impossible.

15. In the polynomial, $f(x) = -x^4 + 21x^2 + 100,$ the leading coefficient is –1 and the degree is 4. Applying the Leading Coefficient Test, we know that even-degree polynomials with negative leading coefficient will fall to the left and to the right. Since the graph falls to the right, we know that the elk population will die out over time.

16. $f(x) = -2(x-1)(x+2)^2(x+5)^3$

$x = 1$, multiplicity 1, the graph crosses the x-axis
$x = -2$, multiplicity 2, the graph touches the x-axis
$x = -5$, multiplicity 5, the graph crosses the x-axis

17.
$$\begin{aligned} f(x) &= x^3 - 5x^2 - 25x + 125 \\ &= x^2(x-5) - 25(x-5) \\ &= (x^2 - 25)(x-5) \\ &= (x+5)(x-5)^2 \end{aligned}$$

$x = -5$, multiplicity 1, the graph crosses the x-axis
$x = 5$, multiplicity 2, the graph touches the x-axis

18. $f(x) = x^3 - 2x - 1$

$f(1) = (1)^3 - 2(1) - 1 = -2$

$f(2) = (2)^3 - 2(2) - 1 = 3$

The sign change shows there is a zero between the given values.

19. $f(x) = x^3 - x^2 - 9x + 9$

a. Since n is odd and $a_n > 0,$ the graph falls to the left and rises to the right.

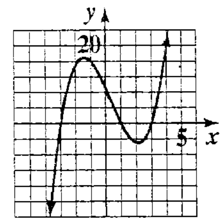
b.
$$\begin{aligned} f(-x) &= (-x)^3 - (-x)^2 - 9(-x) + 9 \\ &= -x^3 - x^2 + 9x + 9 \end{aligned}$$

$f(-x) \neq f(x), f(-x) \neq -f(x)$

no symmetry

c. $f(x) = (x-3)(x+3)(x-1)$

zeros: 3, –3, 1



$f(x) = x^3 - x^2 - 9x + 9$

20. $f(x) = 4x - x^3$

a. Since n is odd and $a_n < 0,$ the graph rises to the left and falls to the right.

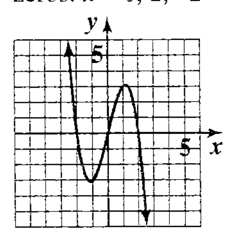
b. $f(-x) = -4x + x^3$

$f(-x) = -f(x)$

origin symmetry

c. $f(x) = x(x^2 - 4) = x(x-2)(x+2)$

zeros: $x = 0, 2, -2$



$f(x) = 4x - x^3$

21. $f(x) = 2x^3 + 3x^2 - 8x - 12$

a. Since h is odd and $a_n > 0,$ the graph falls to the left and rises to the right.

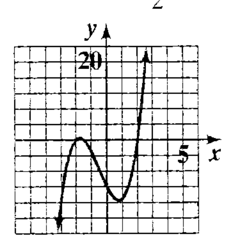
b. $f(-x) = -2x^3 + 3x^2 + 8x - 12$

$f(-x) \neq f(x),\ f(-x) = -f(x)$

no symmetry

c. $f(x) = (x-2)(x+2)(2x+3)$

zeros: $x = 2, -2, -\dfrac{3}{2}$



$f(x) = 2x^3 + 3x^2 - 8x - 12$

22. $g(x) = -x^4 + 25x^2$

a. The graph falls to the left and to the right.

b. $f(-x) = -(-x)^4 + 25(-x)^2$

$= -x^4 + 25x^2 = f(x)$

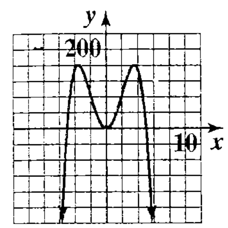
y-axis symmetry

c. $-x^4 + 25x^2 = 0$

$-x^2\left(x^2 - 25\right) = 0$

$-x^2(x-5)(x+5) = 0$

zeros: $x = -5, 0, 5$



$f(x) = -x^4 + 25x^2$

23. $f(x) = -x^4 + 6x^3 - 9x^2$

a. The graph falls to the left and to the right.

b. $f(-x) = -(-x)^4 + 6(-x)^3 - 9(-x)$

$= -x^4 - 6x^3 - 9x^2 f(-x) \neq f(x)$

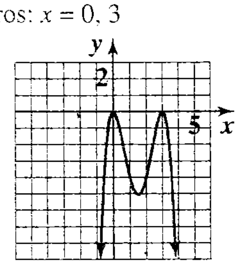
$f(-x) \neq -f(x)$

no symmetry

c. $= -x^2\left(x^2 - 6x + 9\right) = 0$

$-x^2(x-3)(x-3) = 0$

zeros: $x = 0, 3$



$f(x) = -x^4 + 6x^3 - 9x^2$

24. $f(x) = 3x^4 - 15x^3$

a. The graph rises to the left and to the right.

b. $f(-x) = 3(-x)^4 - 15(-x)^3 = 3x^4 + 15x^3$

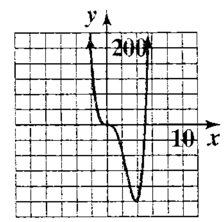
$f(-x) \neq f(x),\ f(-x) \neq -f(x)$

no symmetry

c $3x^4 - 15x^3 = 0$

$3x^3(x-5) = 0$

zeros: $x = 0, 5$



$f(x) = 3x^4 - 15x^3$

25. $f(x) = 2x^2(x-1)^3(x+2)$

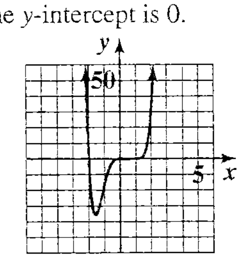
Since $a_n > 0$ and n is even, $f(x)$ rises to the left and the right.

$x = 0, x = 1, x = -2$

The zeros at 1 and -2 have odd multiplicity so $f(x)$ crosses the x-axis at those points. The root at 0 has even multiplicity so $f(x)$ touches the axis at $(0, 0)$

$f(0) = 2(0)^2(0-1)^3(0+2) = 0$

The y-intercept is 0.



$f(x) = 2x^2(x-1)^3(x+2)$

26. $f(x) = -x^3(x+4)^2(x-1)$

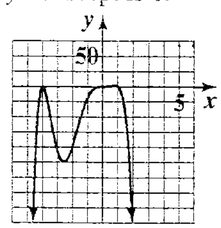
Since $a_n < 0$ and n is even, $f(x)$ falls to the left and the right.

$x = 0, x = -4, x = 1$

The roots at 0 and 1 have odd multiplicity so $f(x)$ crosses the x-axis at those points. The root at -4 has even multiplicity so $f(x)$ touches the axis at $(-4, 0)$

$f(0) = -(0)^3(0+4)^2(0-1) = 0$

The y-intercept is 0.



$f(x) = -x^3(x + 4)^2(x - 1)$

27.

$$
\begin{array}{r}
4x^2 - 7x + 5 \\
x+1 \overline{\big) 4x^3 - 3x^2 - 2x + 1} \\
\underline{4x^3 + 4x^2} \qquad\qquad \\
-7x^2 - 2x \qquad \\
\underline{-7x^2 - 7x} \qquad \\
5x + 1 \\
\underline{5x + 5} \\
-4
\end{array}
$$

Quotient: $4x^2 - 7x + 5 - \dfrac{4}{x+1}$

28.

$$
\begin{array}{r}
2x^2 - 4x + 1 \\
5x-3 \overline{\big) 10x^3 - 26x^2 + 17x - 13} \\
\underline{10x^3 + 6x^2} \qquad\qquad \\
-20x^2 + 17x \qquad \\
\underline{-20x^2 + 12x} \qquad \\
5x - 13 \\
\underline{5x - 3} \\
-10
\end{array}
$$

Quotient: $2x^2 - 4x + 1 - \dfrac{10}{5x-3}$

29.

$$
\begin{array}{r}
2x^2 + 3x - 1 \\
2x^2+1 \overline{\big) 4x^4 + 6x^3 + 3x - 1} \\
\underline{4x^2 + 2x^2} \qquad\qquad \\
6x^3 - 2x^2 + 3x \\
\underline{6x^2 + 3x} \\
-2x^2 - 1 \\
\underline{-2x^2 - 1} \\
0
\end{array}
$$

30. $(3x^4 + 11x^3 - 20x^3 + 7x + 35) \div (x+5)$

-5	3	11	-20	7	35
		-15	20	0	-35
	3	-4	0	7	0

Quotient: $3x^3 - 4x^2 + 7$

31. $(3x^4 - 2x^2 - 10x) \div (x-2)$

2	3	0	-2	-10	0
		6	12	20	20
	3	6	10	10	20

Quotient: $3x^3 + 6x^2 + 10x + 10 + \dfrac{20}{x-2}$

32. $f(x) = 2x^3 - 7x^2 + 9x - 3$

-13	2	-7	9	-3
		-26	429	-5694
	2	-33	438	-5697

Quotient: $f(-13) = -5697$

33. $f(x) = 2x^3 + x^2 - 13x + 6$

2	2	1	-13	6
		4	10	-6
	2	5	-3	0

$$
\begin{aligned}
f(x) &= (x-2)(2x^2 + 5x - 3) \\
&= (x-2)(2x-1)(x+3)
\end{aligned}
$$

Zeros: $x = 2, \dfrac{1}{2}, -3$

34. $x^3 - 17x + 4 = 0$

$$\begin{array}{r|rrrr} 4 & 1 & 0 & -17 & 4 \\ & & 4 & 16 & -4 \\ \hline & 1 & 4 & -1 & 0 \end{array}$$

$(x-4)(x^2+4x-1)=0$

$$x = \frac{-4 \pm \sqrt{16+4}}{2} = \frac{-4 \pm 2\sqrt{5}}{2} = -2 \pm \sqrt{5}$$

The solution set is $\{4, -2+\sqrt{5}, -2-\sqrt{5}\}$.

35. $f(x) = x^4 - 6x^3 + 14x^2 - 14x + 5$

$p: \pm 1, \pm 5$

$q: \pm 1$

$\frac{p}{q}: \pm 1, \pm 5$

36. $f(x) = 3x^5 - 2x^4 - 15x^3 + 10x^2 + 12x - 8$

$p: \pm 1, \pm 2, \pm 4, \pm 8$

$q: \pm 1, \pm 3$

$\frac{p}{q}: \pm 1, \pm 2, \pm 4, \pm 8, \pm\frac{8}{3}, \pm\frac{4}{3}, \pm\frac{2}{3}, \pm\frac{1}{3}$

37. $f(x) = 3x^4 - 2x^3 - 8x + 5$

$f(x)$ has 2 sign variations, so $f(x) = 0$ has 2 or 0 positive solutions.

$f(-x) = 3x^4 + 2x^3 + x + 5$

$f(-x)$ has no sign variations, so $f(x) = 0$ has no negative solutions.

38. $f(x) = 2x^5 - 3x^3 - 5x^2 + 3x - 1$

$f(x)$ has 3 sign variations, so $f(x) = 0$ has 3 or 1 positive real roots.

$f(-x) = -2x^5 + 3x^3 - 5x^2 - 3x - 1$

$f(-x)$ has 2 sign variations, so $f(x) = 0$ has 2 or 0 negative solutions.

39. $f(x) = f(-x) = 2x^4 + 6x^2 + 8$

No sign variations exist for either $f(x)$ or $f(-x)$, so no real roots exist.

40. $f(x) = x^3 + 3x^2 - 4$

a. $p: \pm 1, \pm 2, \pm 4$

$q: \pm 1$

$\frac{p}{q}: \pm 1, \pm 2, \pm 4$

b. 1 sign variation $\Rightarrow$ 1 positive real zero

$f(-x) = -x^3 + 3x^2 - 4$

2 sign variations $\Rightarrow$ 2 or no negative real zeros

c.

$$\begin{array}{r|rrrr} 1 & 1 & 3 & 0 & -4 \\ & & 1 & 4 & -4 \\ \hline & 1 & 4 & 4 & 0 \end{array}$$

1 is a zero.

1, −2 are rational zeros.

d. $(x-1)(x^2+4x+4)=0$

$(x-1)(x+2)^2 = 0$

$x = 1$ or $x = -2$

The solution set is $\{1, -2\}$.

41. $f(x) = 6x^3 + x^2 - 4x + 1$

a. $p: \pm 1$

$q: \pm 1, \pm 2, \pm 3, \pm 6$

$\frac{p}{q}: \pm 1, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{6}$

b. $f(x) = 6x^3 + x^2 - 4x + 1$

2 sign variations; 2 or 0 positive real zeros.

$f(-x) = -6x^3 + x^2 + 4x + 1$

1 sign variation; 1 negative real zero.

c.

$$\begin{array}{r|rrrr} -1 & 6 & 1 & -4 & 1 \\ & & -6 & 5 & -1 \\ \hline & 6 & -5 & 1 & 0 \end{array}$$

−1 is a zero.

$-1, \frac{1}{3}, \frac{1}{2}$ are rational zeros.

d. $6x^3 + x^2 - 4x + 1 = 0$

$(x+1)(6x^2 - 5x + 1) = 0$

$(x+1)(3x-1)(2x-1) = 0$

$x = -1$ or $x = \frac{1}{3}$ or $x = \frac{1}{2}$

The solution set is $\left\{-1, \frac{1}{3}, \frac{1}{2}\right\}$.

42. $f(x)=8x^3-36x^2+46x-15$

a. p: $\pm1, \pm3, \pm5, \pm15$
q: $\pm1, \pm2, \pm4, \pm8$
$\frac{p}{q}: \pm1, \pm3, \pm5, \pm15, \pm\frac{1}{2}, \pm\frac{1}{4}, \pm\frac{1}{8},$
$\pm\frac{3}{2}, \pm\frac{3}{4}, \pm\frac{3}{8}, \pm\frac{5}{2}, \pm\frac{5}{4},$
$\pm\frac{5}{8}, \pm\frac{15}{2}, \pm\frac{15}{4}, \pm\frac{15}{8}$

b. $f(x)=8x^3-36x^2+46x-15$
3 sign variations; 3 or 1 positive real solutions.
$f(-x)=-8x^3-36x^2-46x-15$
0 sign variations; no negative real solutions.

c.
$$\begin{array}{r|rrrr} \frac{1}{2} & 8 & -36 & 46 & -15 \\ & & 4 & -16 & 15 \\ \hline & 8 & -32 & 30 & 0 \end{array}$$

$\frac{1}{2}$ is a zero.
$\frac{1}{2}, \frac{3}{2}, \frac{5}{2}$ are rational zeros.

d.
$$8x^3-36x^2+46x-15=0$$
$$\left(x-\frac{1}{2}\right)(8x^2-32x+30)=0$$
$$2\left(x-\frac{1}{2}\right)(4x-16x+15)=0$$
$$2\left(x-\frac{1}{2}\right)(2x-5)(2x-3)=0$$
$x=\frac{1}{2}$ or $x=\frac{5}{2}$ or $x=\frac{3}{2}$
The solution set is $\left\{\frac{1}{2}, \frac{3}{2}, \frac{5}{2}\right\}$.

43. $2x^3+9x^2-7x+1=0$

a. p: ±1
q: $\pm1, \pm2$
$\frac{p}{q}: \pm1, \pm\frac{1}{2}$

b. $f(x)=2x^3+9x^2-7x+1$
2 sign variations; 2 or 0 positive real zeros.
$f(-x)=-2x^3+9x^2+7x+1$
1 sign variation; 1 negative real zero.

c.
$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & 9 & -7 & 1 \\ & & 1 & 5 & -1 \\ \hline & 2 & 10 & -2 & 0 \end{array}$$

$\frac{1}{2}$ is a rational zero.

d.
$$2x^3+9x^2-7x+1=0$$
$$\left(x-\frac{1}{2}\right)(2x^2+10x-2)=0$$
$$2\left(x-\frac{1}{2}\right)(x^2+5x-1)=0$$
Solving $x^2+5x-1=0$ using the quadratic formula gives $x=\frac{-5\pm\sqrt{29}}{2}$

The solution set is $\left\{\frac{1}{2}, \frac{-5+\sqrt{29}}{2}, \frac{-5-\sqrt{29}}{2}\right\}$.

44. $x^4-x^3-7x^2+x+6=0$

a. $p=\frac{p}{q}: \pm1, \pm2, \pm3, \pm6$

b. $f(x)=x^4-x^3-7x^2+x+6$
2 sign variations; 2 or 0 positive real zeros.
$f(-x)=x^4+x^3-7x^2-x+6$
2 sign variations; 2 or 0 negative real zeros.

c.
$$\begin{array}{r|rrrrr} 1 & 1 & -1 & -7 & 1 & 6 \\ & & 1 & 0 & -7 & -6 \\ \hline & 1 & 0 & -7 & -6 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -1 & 1 & 0 & -7 & -6 \\ & & -1 & 1 & 6 \\ \hline & 1 & -1 & -6 & 0 \end{array}$$

$-2, -1, 1, 3$ are rational zeros.

d.
$$x^4 - x^3 - 7x^2 + x + 6 = 0$$
$$(x-1)(x+1)(x^2 - x + 6) = 0$$
$$(x-1)(x+1)(x-3)(x+2) = 0$$
The solution set is $\{-2, -1, 1, 3\}$.

45. $4x^4 + 7x^2 - 2 = 0$

a. p: $\pm1, \pm2$
q: $\pm1, \pm2, \pm4$
$\frac{p}{q}: \pm1, \pm2, \pm\frac{1}{2}, \pm\frac{1}{4}$

b. $f(x) = 4x^4 + 7x^2 - 2$
1 sign variation; 1 positive real zero.
$f(-x) = 4x^4 + 7x^2 - 2$
1 sign variation; 1 negative real zero.

c.
$$\begin{array}{c|ccccc} \frac{1}{2} & 4 & 0 & 7 & 0 & -2 \\ & & 2 & 1 & 4 & 2 \\ \hline & 4 & 2 & 8 & 4 & 0 \end{array}$$

$$\begin{array}{c|cccc} -\frac{1}{2} & 4 & 2 & 8 & 4 \\ & & -2 & 0 & -4 \\ \hline & 4 & 0 & 8 & 0 \end{array}$$

$-\frac{1}{2}, \frac{1}{2}$ are rational zeros.

d.
$$4x^4 + 7x^2 - 2 = 0$$
$$\left(x - \frac{1}{2}\right)\left(x + \frac{1}{2}\right)(4x^2 + 8) = 0$$
$$4\left(x - \frac{1}{2}\right)\left(x + \frac{1}{2}\right)(x^2 + 2) = 0$$
Solving $x^2 + 2 = 0$ using the quadratic formula gives $x = \pm 2i$

The solution set is $\left\{-\frac{1}{2}, \frac{1}{2}, 2i, -2i\right\}$.

46. $f(x) = 2x^4 + x^3 - 9x^2 - 4x + 4$

a. p: $\pm1, \pm2, \pm4$
q: $\pm1, \pm2$
$\frac{p}{q}: \pm1, \pm2, \pm4, \pm\frac{1}{2}$

b. $f(x) = 2x^4 + x^3 - 9x^2 - 4x + 4$
2 sign variations; 2 or 0 positive real zeros.
$f(-x) = 2x^4 - x^3 - 9x^2 + 4x + 4$
2 sign variations; 2 or 0 negative real zeros.

c.
$$\begin{array}{c|ccccc} 2 & 2 & 1 & -9 & -4 & 4 \\ & & 4 & 10 & 2 & -4 \\ \hline & 2 & 5 & 1 & -2 & 0 \end{array}$$

$$\begin{array}{c|cccc} -1 & 2 & 5 & 1 & -2 \\ & & -2 & -3 & 2 \\ \hline & 2 & 3 & -2 & 0 \end{array}$$

$-2, -1, \frac{1}{2}, 2$ are rational zeros.

d.
$$2x^2 + 3x - 2 = 0$$
$$(2x-1)(x+2) = 0$$
$$x = -2 \quad \text{or} \quad x = \frac{1}{2}$$
The solution set is $\left\{-2, -1, \frac{1}{2}, 2\right\}$.

47.
$$f(x) = a_n(x-2)(x-2+3i)(x-2-3i)$$
$$f(x) = a_n(x-2)(x^2 - 4x + 13)$$
$$f(1) = a_n(1-2)\left[1^2 - 4(1) + 13\right]$$
$$-10 = -10a_n$$
$$a_n = 1$$
$$f(x) = 1(x-2)(x^2 - 4x + 13)$$
$$f(x) = x^3 - 4x^2 + 13x - 2x^2 + 8x - 26$$
$$f(x) = x^3 - 6x^2 + 21x - 26$$

48.
$$f(x) = a_n(x-i)(x+i)(x+3)^2$$
$$f(x) = a_n(x^2+1)(x^2 + 6x + 9)$$
$$f(-1) = a_n\left[(-1)^2 + 1\right]\left[(-1)^2 + 6(-1) + 9\right]$$
$$16 = 8a_n$$
$$a_n = 2$$
$$f(x) = 2(x^2+1)(x^2 + 6x + 9)$$
$$f(x) = 2(x^4 + 6x^3 + 9x^2 + x^2 + 6x + 9)$$
$$f(x) = 2x^4 + 12x^3 + 20x^2 + 12x + 18$$

49. $f(x) = 2x^4 + 3x^3 + 3x - 2$

p: $\pm 1, \pm 2$

q: $\pm 1, \pm 2$

$\frac{p}{q}: \pm 1, \pm 2, \pm \frac{1}{2}$

$$\begin{array}{r|rrrrr} -2 & 2 & 3 & 0 & 3 & -2 \\ & & -4 & 2 & -4 & 2 \\ \hline & 2 & -1 & 2 & -1 & 0 \end{array}$$

$$2x^4 + 3x^3 + 3x - 2 = 0$$
$$(x+2)(2x^3 - x^2 + 2x - 1) = 0$$
$$(x+2)[x^2(2x-1) + (2x-1)] = 0$$
$$(x+2)(2x-1)(x^2+1) = 0$$

$x = -2,\ x = \frac{1}{2}$ or $x = \pm i$

The zeros are -2, $\frac{1}{2}$, $\pm i$.

$f(x) = (x-i)(x+i)(x+2)(2x-1)$

50. $g(x) = x^4 - 6x^3 + x^2 + 24x + 16$

p: $\pm 1, \pm 2, \pm 4, \pm 8, \pm 16$

q: ± 1

$\frac{p}{q}: \pm 1, \pm 2, \pm 4, \pm 8, \pm 16$

$$\begin{array}{r|rrrrr} -1 & 1 & -6 & 1 & 24 & 16 \\ & & -1 & 7 & -8 & -16 \\ \hline & 1 & -7 & 8 & 16 & 0 \end{array}$$

$$x^4 - 6x^3 + x^2 + 24x + 16 = 0$$
$$(x+1)(x^3 - 7x^2 + 8x + 16) = 0$$

$$\begin{array}{r|rrrr} -1 & 1 & -7 & 8 & 16 \\ & & -1 & 8 & -16 \\ \hline & 1 & -8 & 16 & 0 \end{array}$$

$$(x+1)^2(x^2 - 8x + 16) = 0$$
$$(x+1)^2(x-4)^2 = 0$$

$x = -1$ or $x = 4$

$g(x) = (x+1)^2(x-4)^2$

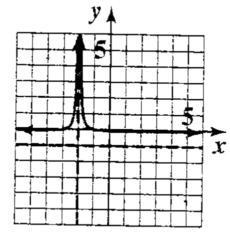
51. 4 real zeros, one with multiplicity two

52. 3 real zeros; 2 nonreal complex zeros

53. 2 real zeros, one with multiplicity two; 2 nonreal complex zeros

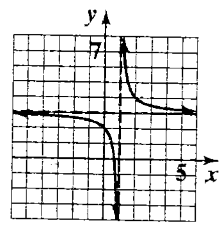
54. 1 real zero; 4 nonreal complex zeros

55. $g(x) = \frac{1}{(x+2)^2} - 1$



$g(x) = \frac{1}{(x+2)^2} - 1$

56. $h(x) = \frac{1}{x-1} + 3$



$h(x) = \frac{1}{x-1} + 3$

57. $f(x) = \frac{2x}{x^2 - 9}$

Symmetry: $f(-x) = -\frac{2x}{x^2-9} = -f(x)$

origin symmetry

x-intercept:

$$0 = \frac{2x}{x^2 - 9}$$
$$2x = 0$$
$$x = 0$$

y-intercept: $y = \frac{2(0)}{0^2 - 9} = 0$

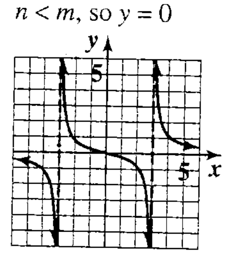
Vertical asymptote:

$$x^2 - 9 = 0$$
$$(x-3)(x+3) = 0$$

$x = 3$ and $x = -3$

Horizontal asymptote:

$n < m$, so $y = 0$



$f(x) = \frac{2x}{x^2 - 9}$

58. $g(x)=\dfrac{2x-4}{x+3}$

Symmetry: $g(-x)=\dfrac{-2x-4}{x+3}$

$g(-x)\neq g(x)$, $g(-x)\neq -g(x)$

No symmetry

x-intercept:

$2x-4=0$

$x=2$

y-intercept: $y=\dfrac{2(0)-4}{(0)+3}=-\dfrac{4}{3}$

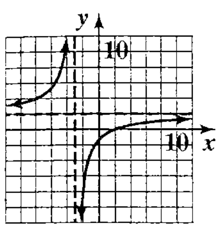
Vertical asymptote:

$x+3=0$

$x=-3$

Horizontal asymptote:

$n=m$, so $y=\dfrac{2}{1}=2$



$f(x)=\dfrac{2x-4}{x+3}$

59. $h(x)=\dfrac{x^2-3x-4}{x^2-x-6}$

Symmetry: $h(-x)=\dfrac{x^2+3x-4}{x^2+x-6}$

$h(-x)\neq h(x)$, $h(-x)\neq -h(x)$

No symmetry

x-intercepts:

$x^2-3x-4=0$

$(x-4)(x+1)$

$x=4 \quad x=-1$

y-intercept: $y=\dfrac{0^2-3(0)-4}{0^2-0-6}=\dfrac{2}{3}$

Vertical asymptotes:

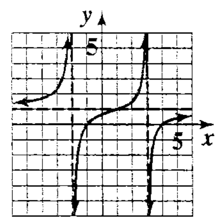
$x^2-x-6=0$

$(x-3)(x+2)=0$

$x=3,-2$

Horizontal asymptote:

$n=m$, so $y=\dfrac{1}{1}=1$



$h(x)=\dfrac{x^2-3x-4}{x^2-x-6}$

60. $r(x)=\dfrac{x^2+4x+3}{(x+2)^2}$

Symmetry: $r(-x)=\dfrac{x^2-4x+3}{(-x+2)^2}$

$r(-x)\neq r(x)$, $r(-x)\neq -r(x)$

No symmetry

x-intercepts:

$x^2+4x+3=0$

$(x+3)(x+1)=0$

$x=-3,-1$

y-intercept: $y=\dfrac{0^2+4(0)+3}{(0+2)^2}=\dfrac{3}{4}$

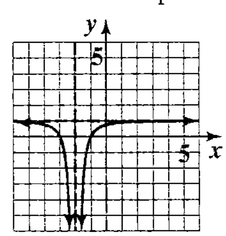
Vertical asymptote:

$x+2=0$

$x=-2$

Horizontal asymptote:

$n=m$, so $y=\dfrac{1}{1}=1$



$r(x)=\dfrac{x^2+4x+3}{(x+4)^2}$

61. $y = \dfrac{x^2}{x+1}$

Symmetry: $f(-x) = \dfrac{x^2}{-x+1}$

$f(-x) \neq f(x), f(-x) \neq -f(x)$

No symmetry

x-intercept:

$x^2 = 0$

$x = 0$

y-intercept: $y = \dfrac{0^2}{0+1} = 0$

Vertical asymptote:

$x + 1 = 0$

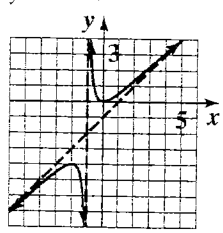
$x = -1$

$n > m$, no horizontal asymptote.

Slant asymptote:

$y = x - 1 + \dfrac{1}{x+1}$

$y = x - 1$



$y = \dfrac{x^2}{x+1}$

62. $y = \dfrac{x^2+2x-3}{x-3}$

Symmetry: $f(-x) = \dfrac{x^2-2x-3}{-x-3}$

$f(-x) \neq f(x), f(-x) \neq -f(x)$

No symmetry

x-intercepts:

$x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$

$x = -3, 1$

y-intercept: $y = \dfrac{0^2+2(0)-3}{0-3} = \dfrac{-3}{-3} = 1$

Vertical asymptote:

$x - 3 = 0$

$x = 3$

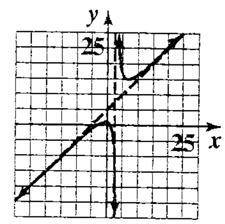
Horizontal asymptote:

$n > m$, so no horizontal asymptote.

Slant asymptote:

$y = x + 5 + \dfrac{12}{x-3}$

$y = x + 5$



$f(x) = \dfrac{x^2 + 2x - 3}{x - 3}$

63. $f(x) = \dfrac{-2x^3}{x^2+1}$

Symmetry: $f(-x) = \dfrac{2}{x^2+1} = -f(x)$

Origin symmetry

x-intercept:

$-2x^3 = 0$

$x = 0$

y-intercept: $y = \dfrac{-2(0)^3}{0^2+1} = \dfrac{0}{1} = 0$

Vertical asymptote:

$x^2 + 1 = 0$

$x^2 = -1$

No vertical asymptote.

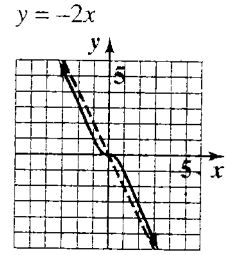
Horizontal asymptote:

$n > m$, so no horizontal asymptote.

Slant asymptote:

$f(x) = -2x + \dfrac{2x}{x^2+1}$

$y = -2x$



$f(x) = \dfrac{-2x^3}{x^2+1}$

64. $g(x)=\dfrac{4x^2-16x+16}{2x-3}$

Symmetry: $g(-x)=\dfrac{4x^2+16x+16}{-2x-3}$

$g(-x)\neq g(x),\ g(-x)\neq -g(x)$

No symmetry

x-intercept:

$4x^2-16x+16=0$

$4(x-2)^2=0$

$x=2$

y-intercept:

$y=\dfrac{4(0)^2-16(0)+16}{2(0)-3}=-\dfrac{16}{3}$

Vertical asymptote:

$2x-3=0$

$x=\dfrac{3}{2}$

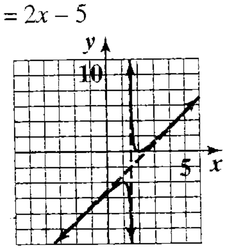
Horizontal asymptote:

$n>m$, so no horizontal asymptote.

Slant asymptote:

$g(x)=2x-5+\dfrac{1}{2x-3}$

$y=2x-5$



$g(x)=\dfrac{4x^2-16x+16}{2x-3}$

65. **a.** $C(x)=50{,}000+25x$

b. $\overline{C}(x)=\dfrac{25x+50{,}000}{x}$

c. $\overline{C}(50)=\dfrac{25(50)+50{,}000}{50}=1025$

When 50 calculators are manufactured, it costs \$1025 to manufacture each.

$\overline{C}(100)=\dfrac{25(100)+50{,}000}{100}=525$

When 100 calculators are manufactured, it costs \$525 to manufacture each.

$\overline{C}(1000)=\dfrac{25(1000)+50{,}000}{1000}=75$

When 1,000 calculators are manufactured, it costs \$75 to manufacture each.

$\overline{C}(100{,}000)=\dfrac{25(100{,}000)+50{,}000}{100{,}000}=25.5$ Wh

en 100,000 calculators are manufactured, it costs \$25.50 to manufacture each.

d. $n=m$, so $y=\dfrac{25}{1}=25$ is the horizontal asymptote. Minimum costs will approach \$25.

66. $f(x)=\dfrac{150x+120}{0.05x+1}$

$n=m$, so $y=\dfrac{150}{0.05}=3000$

The number of fish available in the pond approaches 3000.

67. $P(x)=\dfrac{72{,}900}{100x^2+729}$

$n<m$ so $y=0$

As the number of years of education increases the percentage rate of unemployment approaches zero.

68. **a.** $P(x)=M(x)+F(x)$

$=1.58x+114.4+1.48x+120.6$

$=3.06x+235$

b. $R(x)=\dfrac{M(x)}{P(x)}=\dfrac{1.58x+114.4}{3.06x+235}$

c. $y=\dfrac{1.58}{3.06}\approx 0.52$

Over time, the percentage of men in the U.S. population will approach 52%.

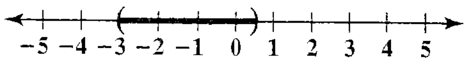
69. $2x^2+5x-3<0$

Solve the related quadratic equation.

$$2x^2+5x-3=0$$
$$(2x-1)(x+3)=0$$

The boundary points are -3 and $\frac{1}{2}$.

Testing each interval gives a solution set of $\left(-3,\frac{1}{2}\right)$



70. $2x^2+9x+4\ge 0$

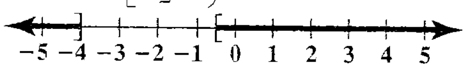
Solve the related quadratic equation.

$$2x^2+9x+4=0$$
$$(2x+1)(x+4)=0$$

The boundary points are -4 and $-\frac{1}{2}$.

Testing each interval gives a solution set of

$(-\infty,-4]\cup\left[-\frac{1}{2},\infty\right)$



71. $x^3+2x^2>3x$

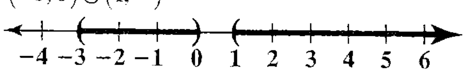
Solve the related equation.

$$x^3+2x^2=3x$$
$$x^3+2x^2-3x=0$$
$$x\left(x^2+2x-3\right)=0$$
$$x(x+3)(x-1)=0$$

The boundary points are -3, 0, and 1.

Testing each interval gives a solution set of

$(-3,0)\cup(1,\infty)$



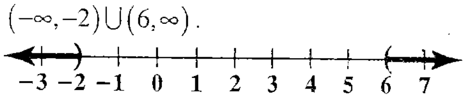
72. $\dfrac{x-6}{x+2}>0$

Find the values of x that make the numerator and denominator zero.

The boundary points are –2 and 6.

Testing each interval gives a solution set of

$(-\infty,-2)\cup(6,\infty)$.



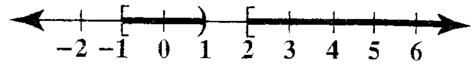
73. $\dfrac{(x+1)(x-2)}{x-1}\ge 0$

Find the values of x that make the numerator and denominator zero.

The boundary points are -1, 1 and 2. We exclude 1 from the solution set, since this would make the denominator zero.

Testing each interval gives a solution set of

$[-1,1)\cup[2,\infty)$.



74. $\dfrac{x+3}{x-4}\le 5$

Express the inequality so that one side is zero.

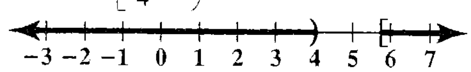
$$\frac{x+3}{x-4}-5\le 0$$
$$\frac{x+3}{x-4}-\frac{5(x-4)}{x-4}\le 0$$
$$\frac{-4x+23}{x-4}\le 0$$

Find the values of x that make the numerator and denominator zero.

The boundary points are 4 and $\frac{23}{4}$. We exclude 4 from the solution set, since this would make the denominator zero.

Testing each interval gives a solution set of

$(-\infty,4)\cup\left[\frac{23}{4},\infty\right)$.



75. **a.** $g(x) = 0.125x^2 + 2.3x + 27$

$g(35) = 0.125(35)^2 + 2.3(35) + 27 \approx 261$

The stopping distance on wet pavement for a motorcycle traveling 35 miles per hour is about 261 feet. This overestimates the distance shown in the graph by 1 foot.

b. $f(x) = 0.125x^2 - 0.8x + 99$

$0.125x^2 - 0.8x + 99 > 267$

$0.125x^2 - 0.8x - 168 > 0$

Solve the related quadratic equation.

$0.125x^2 - 0.8x - 168 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-0.8) \pm \sqrt{(-0.8)^2 - 4(0.125)(-168)}}{2(0.125)}$$

$x = -33.6,\ 40$

Testing each interval gives a solution set of $(-\infty, -33.6) \cup (40, \infty)$.

Thus, speeds exceeding 40 miles per hour on dry pavement will require over 267 feet of stopping distance.

76. $s = -16t^2 + v_0t + s_0$

$32 < -16t^2 + 48t + 0$

$0 < -16t^2 + 48t - 32$

$0 < -16\left(t^2 - 3t + 2\right)$

$0 < -16(t-2)(t-1)$

F	T	F
	1 2	

The projectile's height exceeds 32 feet during the time period from 1 to 2 seconds.

77. $w = ks$

$28 = k \cdot 250$

0.112=k

Thus, $w = 0.112s$.

$w = 0.112(1200) = 134.4$

1200 cubic centimeters of melting snow will produce 134.4 cubic centimeters of water.

78. $d = kt^2$

$144 = k(3)^2$

$k = 16$

$d = 16t^2$

$d = 16(10)^2 = 1{,}600$ ft

79. $p = \dfrac{k}{w}$

$660 = \dfrac{k}{1.6}$

$1056 = k$

Thus, $p = \dfrac{1056}{w}$.

$p = \dfrac{1056}{2.4} = 440$

The pitch is 440 vibrations per second.

80. $l = \dfrac{k}{d^2}$

$28 = \dfrac{k}{8^2}$

$k = 1792$

$l = \dfrac{1792}{d^2}$

$l = \dfrac{1792}{4^2} = 112$ decibels

81. $t = \dfrac{kc}{w}$

$10 = \dfrac{k \cdot 30}{6}$

$10 = 5h$

$h = 2$

$t = \dfrac{2c}{w}$

$t = \dfrac{2(40)}{5} = 16$ hours

82. $V = khB$

$175 = k \cdot 15 \cdot 35$

$k = \dfrac{1}{3}$

$V = \dfrac{1}{3}hB$

$V = \dfrac{1}{3} \cdot 20 \cdot 120 = 800\ \text{ft}^3$

83. **a.** Use $L = \frac{k}{R}$ to find k.

$$L = \frac{k}{R}$$
$$30 = \frac{k}{63}$$
$$63 \cdot 30 = 63 \cdot \frac{k}{63}$$
$$1890 = k$$

Thus, $L = \frac{1890}{R}$.

b. This is an approximate model.

c. $L = \frac{1890}{R}$

$$L = \frac{1890}{27} = 70$$

The average life span of an elephant is 70 years.

Chapter 3 Test

1. $f(x) = (x+1)^2 + 4$

vertex: $(-1, 4)$

axis of symmetry: $x = -1$

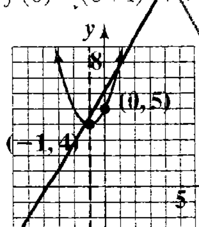
x-intercepts:

$$(x+1)^2 + 4 = 0$$
$$x^2 + 2x + 5 = 0$$
$$x = \frac{-2 \pm \sqrt{4-20}}{2} = -1 \pm 2i$$

no x-intercepts

y-intercept:

$$f(0) = (0+1)^2 + 4 = 5$$



$f(x) = (x+1)^2 + 4$

domain: $(-\infty, \infty)$; range: $[4, \infty)$

2. $f(x) = x^2 - 2x - 3$

$$x = \frac{-b}{2a} = \frac{2}{2} = 1$$
$$f(1) = 1^2 - 2(1) - 3 = -4$$

vertex: $(1, -4)$

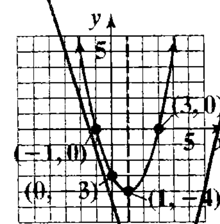
axis of symmetry $x = 1$

x-intercepts:

$$x^2 - 2x - 3 = 0$$
$$(x-3)(x+1) = 0$$
$$x = 3 \text{ or } x = -1$$

y-intercept:

$$f(0) = 0^2 - 2(0) - 3 = -3$$



$f(x) = x^2 - 2x - 3$

domain: $(-\infty, \infty)$; range: $[-4, \infty)$

3. $f(x) = -2x^2 + 12x - 16$

Since the coefficient of x^2 is negative, the graph of $f(x)$ opens down and $f(x)$ has a maximum point.

$$x = \frac{-12}{2(-2)} = 3$$

$$f(3) = -2(3)^2 + 12(3) - 16$$
$$= -18 + 36 - 16$$
$$= 2$$

Maximum point: $(3, 2)$

domain: $(-\infty, \infty)$; range: $(-\infty, 2]$

4. $f(x) = -x^2 + 46x - 360$

$$x = -\frac{b}{2a} = \frac{-46}{-2} = 23$$

23 computers will maximize profit.

$$f(23) = -(23)^2 + 46(23) - 360 = 169$$

Maximum daily profit = $16,900.