1. The function \( G(x) = \begin{cases} x^2, & x < 2 \\ 4 - 2x, & x > 2 \end{cases} \) is not continuous at \( x = 2 \) because:
   
   A. \( G(2) \) is not defined
   
   B. \( \lim_{x \to 2} G(x) \) does not exist
   
   C. \( \lim_{x \to 2} G(x) \neq G(2) \)
   
   D. All of the above
   
   E. None of the above

2. \( \lim_{x \to 0} \left( \frac{1}{x} + \frac{1}{x^2} \right) = \square \)

3. \( \lim_{x \to \infty} \frac{x^3 - 4x + 1}{2x^3 - 5} = \square \)

4. \( \lim_{h \to 0} \frac{\arcsin(1 + h) - \arcsin(1)}{h} = \frac{d}{dx} \arcsin(x) = \frac{1}{\sqrt{1 - x^2}} \)

5. A ladder is 15 feet tall leans against a vertical wall of a home. If the bottom of the ladder is pulled away horizontally from the house at 4 feet/sec, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 9 feet from the wall?

   - Given: \( \frac{dx}{dt} = 4 \)
   - Find: \( \frac{dy}{dt} \) when \( x = 9 \)

   Equation: \( x^2 + y^2 = 15^2 \)

   \( x = 9 \), \( y = 12 \)

   \( \frac{dx}{dt} = 4 \), \( \frac{dy}{dt} = -3 \text{ ft/sec} \)

6. A cone (vertex down) with height 10 inches and radius 5 inches is being filled with water at a constant rate of 2 in\(^3\)/sec. How fast is the surface of the water rising when the depth of the water is 6 inches?

   - Given: \( \frac{dv}{dt} = 2 \)
   - Find: \( \frac{dh}{dt} \) when \( h = 6 \)

   Equation \( V = \frac{1}{3} \pi r^2 h \)

   \( V = \frac{1}{3} \pi \left( \frac{5}{2} \right)^2 \cdot 6 \)

   \( \frac{dv}{dt} = \frac{1}{3} \pi \left( \frac{5}{2} \right)^2 \frac{dh}{dt} = 2 \)

   \( \frac{dh}{dt} = \frac{24}{\pi \cdot 3} \approx 3 \text{ in/sec} \)
7. Find the derivative of \( y = e^{2x} \).
\[
y' = 2e^{2x}
\]

8. Find the derivative of \( y = \ln(\sin x) \).
\[
y' = \cot x
\]

9. Find the derivative of \( y = \frac{e^x - 1}{e^x + 1} \).
\[
y' = \frac{e^x(e^x + 1) - (e^x - 1)e^x}{(e^x + 1)^2}
\]
\[
y' = \frac{2e^x}{(e^x + 1)^2}
\]

10. Find the derivative of \( y = e^{\sin x} \).
\[
y' = e^{\sin x} \cdot \cos x
\]
\[
y' = \cos x \cdot e^{\sin x}
\]

11. Find the equation of the tangent line to the curve \((x^2 + y^2)^2 = 4x^2y\) at the point \((1, 1)\).
\[
y - 1 = 0 (x - 1)
\]
\[
y = 1
\]

12. Find the value of \( c \) guaranteed to exist by Rolle's Theorem for \( f(x) = 2x^2 - 11x + 15 \) in the interval \([\frac{5}{2}, 3]\).
\[
1) \text{ } f \text{ is cont. on } [\frac{5}{2}, 3] \checkmark
\]
\[
2) \text{ } f \text{ is diff. on } (\frac{5}{2}, 3) \checkmark
\]
\[
3) f(\frac{5}{2}) = 0
\]
\[
f(3) = 0 \checkmark
\]
Rolle's Theorem applies
\[
f'(x) = 4x - 11
\]
\[
0 = 4c - 11
\]
\[
c = \frac{11}{4}
\]

13. Find the value of \( c \) guaranteed to exist by the Mean Value Theorem for \( f(x) = (x-1)^3 \) in the interval \([-1, 2]\).
\[
slope \text{ of tangent} \quad f'(x) = 3(x-1)^2
\]
\[
slope \text{ of secant} \quad \frac{f(2) - f(-1)}{2 - (-1)} = 3
\]
\[
3(c-1)^2 = 3
\]
\[
(c-1)^2 = 1
\]
\[
c-1 = \pm 1
\]
\[
c = 2, 0
\]
14. $\int (\sec^2 \theta - \sin \theta) \, d\theta = \tan \theta + \cos \theta + C$

15. $\int (\cos 6x) \, dx = \frac{1}{6} \sin 6x + C$

16. $\int \left( \frac{3}{x^2 - 1} \right) \, dx = \left[ -\frac{3}{x} - x \right]_1^2 = \left[ \left( -3 \frac{3}{2} - 2 \right) - \left( -3 \frac{3}{1} - 1 \right) \right] = \frac{1}{2}$

$u = x^2 + 1$
$x = 1: u = 1^2 + 1 = 2$
$x = -1: u = (-1)^2 + 1 = 2$

$\frac{du}{dx} = 2x \Rightarrow \frac{1}{2} \, du = x \, dx$

$\int \frac{1}{x} (x^2 + 1) \, dx = \frac{1}{2} \int_2^2 u \, du = 0$

18. $\int \frac{e^{-x}}{1 + e^{-x}} \, dx = -\ln \left(1 + e^{-x}\right) + C$

19. $\int 3^x \, dx = \frac{1}{\ln 3} 3^x + C$

$u = \ln x$
$\frac{du}{dx} = \frac{1}{x}$

$x = e^u: u = \ln e^2 = 2$
$x = e: u = \ln e = 1$

20. $\int \frac{1}{x \ln x} \, dx = \int_1^2 \frac{1}{u} \, du$

$= \ln |u|_1^2$

$= \ln 2 - \ln 1$

$= \ln 2$
21. Using the substitution \( u = \sin(2x) \), \( \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin^2(2x) \cos(2x) \, dx \) is equivalent to:

A. \( -2 \int_{\frac{1}{2}}^{1} u^5 \, du \)
B. \( \frac{1}{2} \int_{\frac{1}{2}}^{1} u^5 \, du \)
C. \( \int_{0}^{\sqrt{2}} u^5 \, du \)
D. \( \frac{1}{2} \int_{0}^{\sqrt{2}} u^5 \, du \)
E. \( 2 \int_{0}^{\sqrt{2}} u^5 \, du \)

22. A particle moves along the x-axis with velocity given by \( v(t) = 3t^2 - 4 \) for time \( t \geq 0 \). If the particle is a position \( x = -2 \) at time \( t = 0 \), what is the position of the particle at time \( t = 3 \) ?

\[
x(3) = x(0) + \int_{0}^{3} (3t^2 - 4) \, dt \\
= -2 + \left[ t^3 - 4t \right]_{0}^{3} \\
= -2 + \left[ (27 - 12) - (0 - 0) \right] \\
= 13
\]

23. Find \( \frac{dy}{dx} \) if \( ye^x - x = y^2 \).

\[
\frac{dy}{dx} \cdot e^x + y \cdot e^x - 1 = 2y \frac{dy}{dx} \\
ye^x - 1 = \frac{dy}{dx} (2y - e^x) \\
\frac{dy}{dx} = \frac{y e^x - 1}{2y - e^x}
\]

24. Given \( f(x) = x^3 + 2x + 1 \), find \( (f^{-1})'(1) \).

\[
x = y^3 + 2y + 1 \\
1 = y^3 + 2y + 1 \\
y = 0 \\
\frac{dy}{dx} = \frac{1}{f'(f^{-1}(1))} \\
= \frac{1}{f'(0) + 2} \\
f'(0) = 3(0)^2 + 2 \\
f'(0) = 2 \\
\]

25. Find the derivative of \( y = \arctan \left( \frac{x}{2} \right) \).

\[
y' = \frac{\frac{1}{2}}{1 + \left( \frac{1}{2} \right)^2} = \frac{\frac{1}{2}}{\frac{5}{4}} = \frac{2}{5} + \frac{x^2}{y^2}
\]

26. Find the derivative of \( y = \arccot \left( \sqrt{x} \right) \).

\[
y' = \frac{-\frac{1}{2} \cdot \frac{1}{2\sqrt{x}}}{1 + \left( \frac{1}{2} \right)^2} = \frac{-1}{2\sqrt{x} \left( 1 + x \right)}
\]
27. Find the derivative of $y = \arccos(\cos x)$.

$$y' = -\frac{\sin x}{\sqrt{1 - \cos^2 x}} = \frac{\sin x}{\sin x} = 1$$

28. Find the derivative of $y = \ln(\arcsin x)$.

$$y' = \frac{1}{\arcsin x \sqrt{1 - x^2}} = \frac{1}{\arcsin x \sqrt{1 - x^2}}$$

29. Find the particular solution to the differential equation $xy' - \ln x = 0$ given the initial condition $y(1) = 0$.

$$\frac{y^2}{2} = \frac{(\ln x)_x^2}{2} + C$$

When $x = 1$, $y = 0$, $C = 0$.

$$y^2 = (\ln x)^2$$

30. If $\frac{dy}{dx} = x + 2xy$ and $y(0) = 1$, find $y$ as a function of $x$.

$$\frac{1 + 2y}{1 + 2y} = x + C$$

When $x = 0$, $C = -1$.

$$y = \frac{3e^{x^2} - 1}{2}$$

31. Which of the following is the slope field for the differential equation $y' = \frac{x}{y}$?

A. [Image A]
B. [Image B]
C. [Image C]
D. [Image D]
E. [Image E]
32. Find the volume of the solid generated when the region in the first quadrant bounded by the graphs of \( y = \cos x \), \( y = \sin x \), the line \( x = \frac{\pi}{4} \), and the y-axis is revolved about the x-axis.

\[
R = \cos x - 0 \quad r = \sin x - 0
\]

\[
\pi \int_{0}^{\pi/4} \left[ (\cos x)^2 - (\sin x)^2 \right] \, dx = \pi \int_{0}^{\pi/4} \cos (2x) \, dx = \pi \left[ \frac{1}{2} \sin (2x) \right]_{0}^{\pi/4} = \frac{\pi}{2} \left( 1 - 0 \right) = \frac{\pi}{2}
\]

33. Find the volume of the solid generated when the region in the first quadrant bounded by the graph of \( y = (x+3)^3 \), the line \( x = 2 \), and the x-axis is revolved about the line \( y = -1 \).

\[
R = (x+3)^3 - (-1) \quad r = 0 - (-1)
\]

\[
\pi \int_{2}^{\infty} ((x+3)^3 + 1)^2 - 1 \, dx = \pi \int_{2}^{\infty} (x^6 + 18x^5 + 135x^4 + 542x^3 + 1133x^2 + 1512x + 7383) \, dx = \pi \left[ \frac{x^7}{7} + 3x^6 + 27x^5 + \frac{27}{2}x^4 + 441x^3 + 756x^2 + 7383x \right]_{0}^{\infty} = \pi \left( \frac{27 + 642}{7} - 0 \right) = \frac{778}{7} \pi
\]

34. Let \( R \) be the region bounded by \( y = e^x \), \( y = 2 \), and \( x = 0 \). Find the volume of the solid whose base is bounded by the region \( R \) and the cross sections perpendicular to the x-axis are semicircles.

\[
\text{base} = 2 - e^x
\]

\[
\text{Area semicircle} = \frac{1}{2} \pi r^2 = \frac{1}{2} \pi \left( \frac{2 - e^x}{2} \right)^2 = \frac{\pi}{8} \left( 2 - e^x \right)^2
\]

\[
\pi \int_{0}^{\ln 2} \left( 2 - e^x \right)^2 \, dx = \pi \int_{0}^{\ln 2} (2 - e^x + e^{2x}) \, dx = \pi \left[ 4x - 4e^x + \frac{1}{2}e^{2x} \right]_{0}^{\ln 2} = \pi \left[ (4ln2 - 4e + 1 + 2ln2) - (0 - 4e^{1/2}) \right] = \frac{778}{7} \pi
\]

35. The base of a solid is the region in the first quadrant bounded by the line \( y = -\frac{1}{2}x + 2 \) and the coordinate axes. If every cross section perpendicular to the x-axis is a square, which of the following integrals represents volume of the solid?

A. \( \int_{0}^{2} \left[ -\frac{1}{2}x + 2 \right] \, dx \)

B. \( \int_{0}^{4} \left[ -\frac{1}{2}x + 2 \right] \, dx \)

C. \( \int_{0}^{2} \left[ -\frac{1}{2}x + 2 \right]^2 \, dx \)

D. \( \int_{0}^{4} \left[ -\frac{1}{2}x + 2 \right]^2 \, dx \)

E. None of these