7. $a_n = \frac{(-1)^{n(n+1)/2}}{n^2}$

$a_1 = \frac{(-1)^1}{1^2} = -1$

$a_2 = \frac{(-1)^3}{2^2} = -\frac{1}{4}$

$a_3 = \frac{(-1)^6}{3^2} = \frac{1}{9}$

$a_4 = \frac{(-1)^{10}}{4^2} = \frac{1}{16}$

$a_5 = \frac{(-1)^{15}}{5^2} = -\frac{1}{25}$

9. $a_n = 5 - \frac{1}{n} + \frac{1}{n^2}$

$a_1 = 5 - 1 + 1 = 5$

$a_2 = 5 - \frac{1}{2} + \frac{1}{4} = \frac{19}{4}$

$a_3 = 5 - \frac{1}{3} + \frac{1}{9} = \frac{43}{9}$

$a_4 = 5 - \frac{1}{4} + \frac{1}{16} = \frac{77}{16}$

$a_5 = 5 - \frac{1}{5} + \frac{1}{25} = \frac{121}{25}$

23. $a_n = 3n - 1$

$a_5 = 3(5) - 1 = 14$

$a_6 = 3(6) - 1 = 17$

Add 3 to preceding term.

25. $a_{n+1} = 2a_n, a_1 = 5$

$a_5 = 2(40) = 80$

$a_6 = 2(80) = 160$

Multiply the preceding term by 2.

27. $a_n = \dfrac{3}{(-2)^{n-1}}$

$a_5 = \dfrac{3}{(-2)^4} = \dfrac{3}{16}$

$a_6 = \dfrac{3}{(-2)^5} = -\dfrac{3}{32}$

Multiply the preceding term by $-\dfrac{1}{2}$.

35. $\lim_{n\to\infty} \dfrac{5n^2}{n^2 + 2} = 5$

37. $\lim_{n\to\infty} \dfrac{2n}{\sqrt{n^2 + 1}} = \lim_{n\to\infty} \dfrac{2}{\sqrt{1 + (1/n^2)}} = \dfrac{2}{1} = 2$

39. $\lim_{n\to\infty} \sin\left(\dfrac{1}{n}\right) = 0$

49. $\lim_{n\to\infty} (-1)^n\left(\dfrac{n}{n + 1}\right)$

does not exist (oscillates between -1 and 1), diverges.

51. $\lim_{n\to\infty} \dfrac{3n^2 - n + 4}{2n^2 + 1} = \dfrac{3}{2}$, converges

53. $a_n = \dfrac{1 \cdot 3 \cdot 5 \cdots (2n - 1)}{(2n)^n}$

$= \dfrac{1}{2n} \cdot \dfrac{3}{2n} \cdot \dfrac{5}{2n} \cdots \dfrac{2n - 1}{2n} < \dfrac{1}{2n}$

So, $\lim_{n\to\infty} a_n = 0$, converges.

55. $\lim_{n\to\infty} \dfrac{1 + (-1)^n}{n} = 0$, converges

61. $\lim_{n\to\infty} \dfrac{(n + 1)!}{n!} = \lim_{n\to\infty} (n + 1) = \infty$, diverges

65. $\lim_{n\to\infty} \dfrac{n^p}{e^n} = 0$, converges

$(p > 0, n \geq 2)$

73. $a_n = 3n - 2$

77. $a_n = \dfrac{n + 1}{n + 2}$

81. $a_n = \dfrac{n}{(n + 1)(n + 2)}$

83. $a_n = \dfrac{(-1)^{n-1}}{1 \cdot 3 \cdot 5 \cdots (2n - 1)}$

$= \dfrac{(-1)^{n-1} 2^n n!}{(2n)!}$

85. $a_n = (2n)!, n = 1, 2, 3, \ldots$

87. $a_n = 4 - \dfrac{1}{n} < 4 - \dfrac{1}{n + 1} = a_{n+1},$

Monotonic; $|a_n| < 4$, bounded

89. $\dfrac{n}{2^{n+2}} \overset{?}{\geq} \dfrac{n + 1}{2^{(n+1)+2}}$

$2^{n+3} n \overset{?}{\geq} 2^{n+2}(n + 1)$

$2n \overset{?}{\geq} n + 1$

$n \geq 1$

So, $n \geq 1$

$2n \geq n + 1$

$2^{n+3} n \geq 2^{n+2}(n + 1)$

$\dfrac{n}{2^{n+2}} \geq \dfrac{n + 1}{2^{(n+1)+2}}$

$a_n \geq a_{n+1}.$

Monotonic; $|a_n| \leq \dfrac{1}{8}$, bounded

91. $a_n = (-1)^n\left(\frac{1}{n}\right)$

$a_1 = -1$

$a_2 = \frac{1}{2}$

$a_3 = -\frac{1}{3}$

Not monotonic; $|a_n| \le 1$, bounded

93. $a_n = \left(\frac{2}{3}\right)^n > \left(\frac{2}{3}\right)^{n+1} = a_{n+1}$

Monotonic; $|a_n| \le \frac{2}{3}$, bounded

95. $a_n = \sin\left(\frac{n\pi}{6}\right)$

$a_1 = 0.500$

$a_2 = 0.8660$

$a_3 = 1.000$

$a_4 = 0.8660$

Not monotonic; $|a_n| \le 1$, bounded

97. $a_n = \frac{\cos n}{n}$

$a_1 = 0.5403$

$a_2 = -0.2081$

$a_3 = -0.3230$

$a_4 = -0.1634$

Not monotonic; $|a_n| \le 1$, bounded

115. $a_n = \frac{10^n}{n!}$

(a) $a_9 = a_{10} = \frac{10^9}{9!} = \frac{1{,}000{,}000{,}000}{362{,}880} = \frac{1{,}562{,}500}{567}$

(b) Decreasing

(c) Factorials increase more rapidly than exponentials.

117. $a_n = \sqrt[n]{n} = n^{1/n}$

$a_1 = 1^{1/1} = 1$

$a_2 = \sqrt{2} \approx 1.4142$

$a_3 = \sqrt[3]{3} \approx 1.4422$

$a_4 = \sqrt[4]{4} \approx 1.4142$

$a_5 = \sqrt[5]{5} \approx 1.3797$

$a_6 = \sqrt[6]{6} \approx 1.3480$

Let $y = \lim_{n\to\infty} n^{1/n}$.

$$\ln y = \lim_{n\to\infty}\left(\frac{1}{n}\ln n\right) = \lim_{n\to\infty}\frac{\ln n}{n} = \lim_{n\to\infty}\frac{1/n}{n} = 0$$

Because $\ln y = 0$, you have $y = e^0 = 1$. Therefore,

$\lim_{n\to\infty} \sqrt[n]{n} = 1.$

125. $a_{n+2} = a_n + a_{n+1}$

(a)

$a_1 = 1$	$a_7 = 8 + 5 = 13$
$a_2 = 1$	$a_8 = 13 + 8 = 21$
$a_3 = 1 + 1 = 2$	$a_9 = 21 + 13 = 34$
$a_4 = 2 + 1 = 3$	$a_{10} = 34 + 21 = 55$
$a_5 = 3 + 2 = 5$	$a_{11} = 55 + 34 = 89$
$a_6 = 5 + 3 = 8$	$a_{12} = 89 + 55 = 144$

(b) $b_n = \dfrac{a_{n+1}}{a_n}, n \geq 1$

$b_1 = \dfrac{1}{1} = 1$	$b_6 = \dfrac{13}{8} = 1.625$
$b_2 = \dfrac{2}{1} = 2$	$b_7 = \dfrac{21}{13} \approx 1.6154$
$b_3 = \dfrac{3}{2} = 1.5$	$b_8 = \dfrac{34}{21} \approx 1.6190$
$b_4 = \dfrac{5}{3} \approx 1.6667$	$b_9 = \dfrac{55}{34} \approx 1.6176$
$b_5 = \dfrac{8}{5} = 1.6$	$b_{10} = \dfrac{89}{55} \approx 1.6182$

(c) $$1 + \frac{1}{b_{n-1}} = 1 + \frac{1}{a_n/a_{n-1}}$$

$$= 1 + \frac{a_{n-1}}{a_n} = \frac{a_n + a_{n-1}}{a_n} = \frac{a_{n+1}}{a_n} = b_n$$

(d) If $\lim_{n\to\infty} b_n = \rho$, then $\lim_{n\to\infty}\left(1 + \dfrac{1}{b_{n-1}}\right) = \rho$.

Because $\lim_{n\to\infty} b_n = \lim_{n\to\infty} b_{n-1}$, you have

$1 + (1/\rho) = \rho$.

$$\rho + 1 = \rho^2$$

$$0 = \rho^2 - \rho - 1$$

$$\rho = \frac{1 \pm \sqrt{1 + 4}}{2} = \frac{1 \pm \sqrt{5}}{2}$$

Because a_n, and therefore b_n, is positive,

$$\rho = \frac{1 + \sqrt{5}}{2} \approx 1.6180.$$