2. $S_1 = \frac{1}{6} \approx 0.1667$

$S_2 = \frac{1}{6} + \frac{1}{6} \approx 0.3333$

$S_3 = \frac{1}{6} + \frac{1}{6} + \frac{3}{20} \approx 0.4833$

$S_4 = \frac{1}{6} + \frac{1}{6} + \frac{3}{20} + \frac{2}{15} \approx 0.6167$

$S_5 = \frac{1}{6} + \frac{1}{6} + \frac{3}{20} + \frac{2}{15} + \frac{5}{42} \approx 0.7357$

5. $S_1 = 3$

$S_2 = 3 + \frac{3}{2} = 4.5$

$S_3 = 3 + \frac{3}{2} + \frac{3}{4} = 5.250$

$S_4 = 3 + \frac{3}{2} + \frac{3}{4} + \frac{3}{8} = 5.625$

$S_5 = 3 + \frac{3}{2} + \frac{3}{4} + \frac{3}{8} + \frac{3}{16} = 5.8125$

7. $a_n = \dfrac{n+1}{n}$

$\{a_n\} = \left\{\frac{2}{1}, \frac{3}{2}, \frac{4}{3}, \ldots\right\}$ converges to 1

$\sum_{n=1}^{\infty} a_n = \frac{2}{1} + \frac{3}{2} + \frac{4}{3} + \cdots$ diverges

9. $\sum_{n=0}^{\infty} \left(\frac{7}{6}\right)^n$

Geometric series

$r = \frac{7}{6} > 1$

Diverges by Theorem 9.6

13. $\sum_{n=1}^{\infty} \dfrac{n}{n+1}$

$\lim_{n\to\infty} \dfrac{n}{n+1} = 1 \neq 0$

Diverges by Theorem 9.9

15. $\sum_{n=1}^{\infty} \dfrac{n^2}{n^2+1}$

$\lim_{n\to\infty} \dfrac{n^2}{n^2+1} = 1 \neq 0$

Diverges by Theorem 9.9

17. $\sum_{n=1}^{\infty} \frac{2^n + 1}{2^{n+1}}$

$$\lim_{n\to\infty} \frac{2^n + 1}{2^{n+1}} = \lim_{n\to\infty} \frac{1 + 2^{-n}}{2} = \frac{1}{2} \neq 0$$

Diverges by Theorem 9.9

25. $\sum_{n=0}^{\infty} \left(\frac{5}{6}\right)^n$

Geometric series with $r = \frac{5}{6} < 1$

Converges by Theorem 9.6

27. $\sum_{n=0}^{\infty} (0.9)^n$

Geometric series with $r = 0.9 < 1$

Converges by Theorem 9.6

29. $\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1}\right) = \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{5}\right) + \cdots, \quad S_n = 1 - \frac{1}{n+1}$

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \lim_{n\to\infty} S_n = \lim_{n\to\infty} \left(1 - \frac{1}{n+1}\right) = 1$$

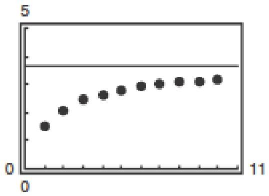
31. (a) $\sum_{n=1}^{\infty} \frac{6}{n(n+3)} = 2\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+3}\right) = 2\left[\left(1 - \frac{1}{4}\right) + \left(\frac{1}{2} - \frac{1}{5}\right) + \left(\frac{1}{3} - \frac{1}{6}\right) + \left(\frac{1}{4} - \frac{1}{7}\right) + \cdots\right]$

$$\left(S_n = 2\left[1 + \frac{1}{2} + \frac{1}{3} - \left(\frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3}\right)\right]\right) = 2\left(1 + \frac{1}{2} + \frac{1}{3}\right) = \frac{11}{3} \approx 3.667$$

(b)

n	5	10	20	50	100
S_n	2.7976	3.1643	3.3936	3.5513	3.6078

(c)



(d) The terms of the series decrease in magnitude slowly. So, the sequence of partial sums approaches the sum slowly.

37. $\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n = \frac{1}{1 - (1/2)} = 2$

41. $\displaystyle\sum_{n=2}^{\infty}\frac{1}{n^2-1}=\sum_{n=2}^{\infty}\left(\frac{1/2}{n-1}-\frac{1/2}{n+1}\right)=\frac{1}{2}\sum_{n=2}^{\infty}\left(\frac{1}{n-1}-\frac{1}{n+1}\right)$

$$S_n=\frac{1}{2}\left[\left(1-\frac{1}{3}\right)+\left(\frac{1}{2}-\frac{1}{4}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)+\cdots+\left(\frac{1}{n-2}-\frac{1}{n}\right)+\left(\frac{1}{n-1}-\frac{1}{n+1}\right)\right]=\frac{1}{2}\left(1+\frac{1}{2}-\frac{1}{n}-\frac{1}{n+1}\right)$$

$$\sum_{n=2}^{\infty}\frac{1}{n^2-1}=\lim_{n\to\infty}S_n=\lim_{n\to\infty}\frac{1}{2}\left(1+\frac{1}{2}-\frac{1}{n}-\frac{1}{n+1}\right)=\frac{3}{4}$$

43. $\displaystyle\sum_{n=1}^{\infty}\frac{8}{(n+1)(n+2)}=8\sum_{n=1}^{\infty}\left(\frac{1}{n+1}-\frac{1}{n+2}\right)$

$$S_n=8\left[\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\left(\frac{1}{4}-\frac{1}{5}\right)+\cdots+\left(\frac{1}{n+1}-\frac{1}{n+2}\right)\right]=8\left(\frac{1}{2}-\frac{1}{n+2}\right)$$

$$\sum_{n=1}^{\infty}\frac{8}{(n+1)(n+2)}=\lim_{n\to\infty}S_n=\lim_{n\to\infty}8\left(\frac{1}{2}-\frac{1}{n+2}\right)=4$$

45. $\displaystyle\sum_{n=0}^{\infty}\left(\frac{1}{10}\right)^n=\frac{1}{1-(1/10)}=\frac{10}{9}$

49. $\displaystyle\sum_{n=0}^{\infty}\left(\frac{1}{2^n}-\frac{1}{3^n}\right)=\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n-\sum_{n=0}^{\infty}\left(\frac{1}{3}\right)^n$

$$=\frac{1}{1-(1/2)}-\frac{1}{1-(1/3)}=2-\frac{3}{2}=\frac{1}{2}$$

51. Note that $\sin(1)\approx 0.8415<1$. The series $\displaystyle\sum_{n=1}^{\infty}[\sin(1)]^n$

is geometric with $r=\sin(1)<1$. So,

$$\sum_{n=1}^{\infty}[\sin(1)]^n=\sin(1)\sum_{n=0}^{\infty}[\sin(1)]^n=\frac{\sin(1)}{1-\sin(1)}\approx 5.3080.$$

52. $\displaystyle S_n=\sum_{k=1}^{n}\frac{1}{9k^2+3k-2}=\sum_{k=1}^{n}\frac{1}{(3k-1)(3k+2)}$

$$=\sum_{k=1}^{n}\left[\frac{1}{9k-3}-\frac{1}{9k+6}\right]=\frac{1}{3}\sum_{k=1}^{n}\left[\frac{1}{3k-1}-\frac{1}{3k+2}\right]$$

$$=\frac{1}{3}\left[\left(\frac{1}{2}-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{8}\right)+\left(\frac{1}{8}-\frac{1}{11}\right)+\cdots+\left(\frac{1}{3n-1}-\frac{1}{3n+2}\right)\right]=\frac{1}{3}\left(\frac{1}{2}-\frac{1}{3n+2}\right)$$

$$\lim_{n\to\infty}S_n=\lim_{n\to\infty}\frac{1}{3}\left(\frac{1}{2}-\frac{1}{3n+2}\right)=\frac{1}{6}$$

57. (a) $0.0\overline{75} = \sum_{n=0}^{\infty} \frac{3}{40}\left(\frac{1}{100}\right)^n$

(b) Geometric series with $a = \frac{3}{40}$ and $r = \frac{1}{100}$

$$S = \frac{a}{1-r} = \frac{3/40}{99/100} = \frac{5}{66}$$

59. $\sum_{n=0}^{\infty} (1.075)^n$

Geometric series with $r = 1.075$

Diverges by Theorem 9.6

61. $\sum_{n=1}^{\infty} \frac{n+10}{10n+1}$

$$\lim_{n\to\infty} \frac{n+10}{10n+1} = \frac{1}{10} \neq 0$$

Diverges by Theorem 9.9

63. $\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+2}\right)$

$$S_n = \left(1 - \frac{1}{3}\right) + \left(\frac{1}{2} - \frac{1}{4}\right) + \left(\frac{1}{3} - \frac{1}{5}\right) + \cdots + \left(\frac{1}{n-1} - \frac{1}{n+1}\right) + \left(\frac{1}{n} - \frac{1}{n+2}\right) = 1 + \frac{1}{2} - \frac{1}{n+1} - \frac{1}{n+2}$$

$$\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+2}\right) = \lim_{n\to\infty} S_n = \lim_{n\to\infty} \left(1 + \frac{1}{2} - \frac{1}{n+1} - \frac{1}{n+2}\right) = \frac{3}{2}, \text{ converges}$$

65. $\sum_{n=1}^{\infty} \frac{1}{n(n+3)} = \frac{1}{3}\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+3}\right)$

$$S_n = \frac{1}{3}\left[\left(\frac{1}{1} - \frac{1}{4}\right) + \left(\frac{1}{2} - \frac{1}{5}\right) + \left(\frac{1}{3} - \frac{1}{6}\right) + \left(\frac{1}{4} - \frac{1}{7}\right) + \cdots + \left(\frac{1}{n-2} - \frac{1}{n+1}\right) - \left(\frac{1}{n-1} - \frac{1}{n+2}\right) - \left(\frac{1}{n} - \frac{1}{n+3}\right)\right]$$

$$= \frac{1}{3}\left(1 + \frac{1}{2} + \frac{1}{3} - \frac{1}{n+1} - \frac{1}{n+2} - \frac{1}{n+3}\right)$$

$$\sum_{n=1}^{\infty} \frac{1}{n(n+3)} = \lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{1}{3}\left(1 + \frac{1}{2} + \frac{1}{3} - \frac{1}{n+1} - \frac{1}{n+2} - \frac{1}{n+3}\right) = \frac{1}{3}\left(\frac{11}{6}\right) = \frac{11}{8}, \text{ converges}$$

67. $\sum_{n=1}^{\infty} \frac{3n - 1}{2n + 1}$

$\lim_{n\to\infty} \frac{3n - 1}{2n + 1} = \frac{3}{2} \neq 0$

Diverges by Theorem 9.9

69. $\sum_{n=0}^{\infty} \frac{4}{2^n} = 4\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n$

Geometric series with $r = \frac{1}{2}$

Converges by Theorem 9.6

71. Because $n > \ln(n)$, the terms $a_n = \frac{n}{\ln(n)}$ do not approach 0 as $n \to \infty$. So, the series $\sum_{n=2}^{\infty} \frac{n}{\ln(n)}$ diverges.

73. For $k \neq 0$,

$$\lim_{n\to\infty} \left(1 + \frac{k}{n}\right)^n = \lim_{n\to\infty} \left[\left(1 + \frac{k}{n}\right)^{n/k}\right]^k$$
$$= e^k \neq 0.$$

For $k = 0, \lim_{n\to\infty} (1 + 0)^n = 1 \neq 0$.

So, $\sum_{n=1}^{\infty} \left[1 + \frac{k}{n}\right]^n$ diverges.

75. $\lim_{n\to\infty} \arctan n = \frac{\pi}{2} \neq 0$

So, $\sum_{n=1}^{\infty} \arctan n$ diverges.

83. $\sum_{n=1}^{\infty} \frac{x^n}{2^n} = \sum_{n=1}^{\infty} \left(\frac{x}{2}\right)^n = \frac{x}{2}\sum_{n=0}^{\infty} \left(\frac{x}{2}\right)^n$

Geometric series: converges for $\left|\frac{x}{2}\right| < 1$ or $|x| < 2$

$$f(x) = \frac{x}{2}\sum_{n=0}^{\infty} \left(\frac{x}{2}\right)^n$$
$$= \frac{x}{2}\frac{1}{1 - (x/2)} = \frac{x}{2}\frac{2}{2 - x} = \frac{x}{2 - x}, \qquad |x| < 2$$

85. $\sum_{n=1}^{\infty}(x-1)^n = (x-1)\sum_{n=0}^{\infty}(x-1)^n$

Geometric series: converges for

$|x-1| < 1 \Rightarrow 0 < x < 2$

$$f(x) = (x-1)\sum_{n=0}^{\infty}(x-1)^n$$

$$= (x-1)\frac{1}{1-(x-1)} = \frac{x-1}{2-x}, \qquad 0 < x < 2$$

87. $\sum_{n=0}^{\infty}(-1)^n x^n = \sum_{n=0}^{\infty}(-x)^n$

Geometric series: converges for

$|-x| < 1 \Rightarrow |x| < 1 \Rightarrow -1 < x < 1$

$$f(x) = \sum_{n=0}^{\infty}(-x)^n = \frac{1}{1+x}, \quad -1 < x < 1$$

89. $\sum_{n=0}^{\infty}\left(\frac{1}{x}\right)^n$

Geometric series: converges if $\left|\frac{1}{x}\right| < 1$

$\Rightarrow |x| > 1 \Rightarrow x < -1 \text{ or } x > 1$

$$f(x) = \sum_{n=0}^{\infty}\left(\frac{1}{x}\right)^n = \frac{1}{1-(1/x)} = \frac{x}{x-1}, \; x > 1 \text{ or } x < -1$$

103. $\sum_{i=0}^{\infty} 200(0.75)^i = 800$ million dollars

105. $D_1 = 16$

$$D_2 = \underbrace{0.81(16)}_{\text{up}} + \underbrace{0.81(16)}_{\text{down}} = 32(0.81)$$

$$D_3 = 16(0.81)^2 + 16(0.81)^2 = 32(0.81)^2$$

$$\vdots$$

$$D = 16 + 32(0.81) + 32(0.81)^2 + \cdots$$

$$= -16 + \sum_{n=0}^{\infty} 32(0.81)^n = -16 + \frac{32}{1-0.81}$$

$$\approx 152.42 \text{ feet}$$

115. $w = \sum_{i=0}^{n-1} 0.01(2)^i = \frac{0.01(1 - 2^n)}{1 - 2} = 0.01(2^n - 1)$

(a) When $n = 29$: $w = \$5,368,709.11$

(b) When $n = 30$: $w = \$10,737,418.23$

(c) When $n = 31$: $w = \$21,474,836.47$