**3.** $0 < \frac{1}{n^2 + 1} < \frac{1}{n^2}$

Therefore,

$$\sum_{n=1}^{\infty} \frac{1}{n^2 + 1}$$

converges by comparison with the convergent $p$-series

$$\sum_{n=1}^{\infty} \frac{1}{n^2}.$$

**5.** $\frac{1}{2n - 1} > \frac{1}{2n} > 0$ for $n \geq 1$

Therefore,

$$\sum_{n=1}^{\infty} \frac{1}{2n - 1}$$

diverges by comparison with the divergent $p$-series $\frac{1}{2}\sum_{n=1}^{\infty} \frac{1}{n}$.

**7.** $\frac{1}{4^n + 1} < \frac{1}{4^n}$

Therefore,

$$\sum_{n=0}^{\infty} \frac{1}{4^n + 1}$$

converges by comparison with the convergent geometric series

$$\sum_{n=0}^{\infty} \frac{1}{4^n}.$$

**9.** For $n \geq 3$, $\frac{\ln n}{n + 1} > \frac{1}{n + 1} > 0$.

Therefore,

$$\sum_{n=1}^{\infty} \frac{\ln n}{n + 1}$$

diverges by comparison with the divergent series

$$\sum_{n=1}^{\infty} \frac{1}{n + 1}.$$

**Note:** $\sum_{n=1}^{\infty} \frac{1}{n + 1}$ diverges by the Integral Test.

**11.** For $n > 3, \frac{1}{n^2} > \frac{1}{n!} > 0.$

Therefore,

$$\sum_{n=0}^{\infty} \frac{1}{n!}$$

converges by comparison with the convergent $p$-series

$$\sum_{n=1}^{\infty} \frac{1}{n^2}.$$

**13.** $0 < \frac{1}{e^{n^2}} \leq \frac{1}{e^n}$

Therefore,

$$\sum_{n=0}^{\infty} \frac{1}{e^{n^2}}$$

converges by comparison with the convergent geometric series

$$\sum_{n=0}^{\infty} \left(\frac{1}{e}\right)^n.$$

**15.** $\lim_{n\to\infty} \frac{n/(n^2+1)}{1/n} = \lim_{n\to\infty} \frac{n^2}{n^2+1} = 1$

Therefore,

$$\sum_{n=1}^{\infty} \frac{n}{n^2+1}$$

diverges by a limit comparison with the divergent $p$-series

$$\sum_{n=1}^{\infty} \frac{1}{n}.$$

**17.** $\lim_{n\to\infty} \frac{1/\sqrt{n^2+1}}{1/n} = \lim_{n\to\infty} \frac{n}{\sqrt{n^2+1}} = 1$

Therefore,

$$\sum_{n=0}^{\infty} \frac{1}{\sqrt{n^2+1}}$$

diverges by a limit comparison with the divergent $p$-series

$$\sum_{n=1}^{\infty} \frac{1}{n}.$$

**19.** $\lim_{n\to\infty} \frac{\frac{2n^2 - 1}{3n^5 + 2n + 1}}{1/n^3} = \lim_{n\to\infty} \frac{2n^5 - n^3}{3n^5 + 2n + 1} = \frac{2}{3}$

Therefore,

$$\sum_{n=1}^{\infty} \frac{2n^2 - 1}{3n^5 + 2n + 1}$$

converges by a limit comparison with the convergent $p$-series

$$\sum_{n=1}^{\infty} \frac{1}{n^3}.$$

**21.** $\lim_{n\to\infty} \frac{(n + 3)/n(n^2 + 4)}{1/n^2} = \lim_{n\to\infty} \frac{(n + 3)n^2}{n(n^2 + 4)} = 1$

Therefore,

$$\sum_{n=1}^{\infty} \frac{n + 3}{n(n^2 + 4)}$$

converges by a limit comparison with the convergent $p$-series

$$\sum_{n=1}^{\infty} \frac{1}{n^2}.$$

**23.** $\lim_{n\to\infty} \frac{1/\left(n\sqrt{n^2 + 1}\right)}{1/n^2} = \lim_{n\to\infty} \frac{n^2}{n\sqrt{n^2 + 1}} = 1$

Therefore,

$$\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n^2 + 1}}$$

converges by a limit comparison with the convergent $p$-series

$$\sum_{n=1}^{\infty} \frac{1}{n^2}.$$

**25.** $\lim_{n\to\infty} \frac{\left(n^{k-1}\right)/\left(n^k + 1\right)}{1/n} = \lim_{n\to\infty} \frac{n^k}{n^k + 1} = 1$

Therefore,

$$\sum_{n=1}^{\infty} \frac{n^{k-1}}{n^k + 1}$$

diverges by a limit comparison with the divergent $p$-series

$$\sum_{n=1}^{\infty} \frac{1}{n}.$$

**27.** $\lim_{n\to\infty} \frac{\sin(1/n)}{1/n} = \lim_{n\to\infty} \frac{\left(-1/n^2\right)\cos(1/n)}{-1/n^2} = \lim_{n\to\infty} \cos\left(\frac{1}{n}\right) = 1$

Therefore,

$$\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$$

diverges by a limit comparison with the divergent $p$-series

$$\sum_{n=1}^{\infty} \frac{1}{n}.$$

**29.** $\sum_{n=1}^{\infty} \frac{\sqrt[1/3]{n}}{n} = \sum_{n=1}^{\infty} \frac{1}{n^{2/3}}$

Diverges;

$p$-series with $p = \frac{2}{3}$

**31.** $\sum_{n=1}^{\infty} \frac{1}{5^n + 1}$

Converges;

Direct comparison with convergent geometric series

$$\sum_{n=1}^{\infty} \left(\frac{1}{5}\right)^n$$

**33.** $\sum_{n=1}^{\infty} \frac{2n}{3n - 2}$

Diverges; $n^{\text{th}}$-Term Test

$$\lim_{n\to\infty} \frac{2n}{3n - 2} = \frac{2}{3} \neq 0$$

**35.** $\sum_{n=1}^{\infty} \frac{n}{\left(n^2 + 1\right)^2}$

Converges; Integral Test