11. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n+1}$

$$a_{n+1} = \frac{1}{n+2} < \frac{1}{n+1} = a_n$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{n+1} = 0$$

Converges by Theorem 9.14

15. $\sum_{n=1}^{\infty} \frac{(-1)^n(5n-1)}{4n+1}$

$$\lim_{n\to\infty} \frac{5n-1}{4n+1} = \frac{5}{4}$$

Diverges by nth-Term test

19. $\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{n^2+5}$

$$\lim_{n\to\infty} \frac{n^2}{n^2+5} = 1$$

Diverges by nth-Term test

23. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(n+1)}{\ln(n+1)}$

$$\lim_{n\to\infty} \frac{n+1}{\ln(n+1)} = \lim_{n\to\infty} \frac{1}{1/(n+1)} = \lim_{n\to\infty} (n+1) = \infty$$

Diverges by the nth-Term Test

27. $\sum_{n=1}^{\infty} \cos n\pi = \sum_{n=1}^{\infty} (-1)^n$

Diverges by the nth-Term Test

31. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\sqrt{n}}{n+2}$

$$a_{n+1} = \frac{\sqrt{n+1}}{(n+1)+2} < \frac{\sqrt{n}}{n+2} \text{ for } n \geq 2$$

$$\lim_{n\to\infty} \frac{\sqrt{n}}{n+2} = 0$$

Converges by Theorem 9.14

35. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(2)}{e^n - e^{-n}} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(2e^n)}{e^{2n} - 1}$

Let $f(x) = \frac{2e^x}{e^{2x} - 1}$. Then

$$f'(x) = \frac{-2e^x(e^{2x} + 1)}{(e^{2x} - 1)^2} < 0.$$

So, $f(x)$ is decreasing. Therefore, $a_{n+1} < a_n$, and

$$\lim_{n\to\infty} \frac{2e^n}{e^{2n} - 1} = \lim_{n\to\infty} \frac{2e^n}{2e^{2n}} = \lim_{n\to\infty} \frac{1}{e^n} = 0.$$

The series converges by Theorem 9.14.

37. $S_6 = \sum_{n=0}^{5} \frac{2(-1)^n}{n!} \approx 0.7333$

$0.7333 - 0.002778 \le S \le 0.7333 + 0.002778$

$|R_6| = |S - S_6| \le a_7 = \frac{2}{6!} = 0.002778$

$0.7305 \le S \le 0.7361$

39. $S_6 = \sum_{n=1}^{6} \frac{3(-1)^{n+1}}{n^2} = 2.4325$

$2.4325 - 0.0612 \le S \le 2.4325 + 0.0612$

$|R_6| = |S - S_6| \le a_7 = \frac{3}{49} \approx 0.0612$

$2.3713 \le S \le 2.4937$

41. $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$

(a) By Theorem 9.15,

$$|R_N| \le a_{N+1} = \frac{1}{(N + 1)!} < 0.001.$$

This inequality is valid when $N = 6$.

(b) Approximate the series by

$$\sum_{n=0}^{6} \frac{(-1)^n}{n!} = 1 - 1 + \frac{1}{2} - \frac{1}{6} + \frac{1}{24} - \frac{1}{120} + \frac{1}{720}$$
$$\approx 0.368.$$

(7 terms. Note that the sum begins with $n = 0$.)

43. $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}$

(a) By Theorem 9.15,

$$\left|R_N\right| \leq a_{N+1} = \frac{1}{[2(N+1)+1]!} < 0.001.$$

This inequality is valid when $N = 2$.

(b) Approximate the series by

$$\sum_{n=0}^{2} \frac{(-1)^n}{(2n+1)!} = 1 - \frac{1}{6} + \frac{1}{120} \approx 0.842.$$

(3 terms. Note that the sum begins with $n = 0$.)

45. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n!}$

(a) By Theorem 9.15,

$$\left|R_N\right| \leq a_{N+1} = \frac{1}{N+1} < 0.001.$$

This inequality is valid when $N = 1000$.

(b) Approximate the series by

$$\sum_{n=1}^{1000} \frac{(-1)^{n+1}}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots - \frac{1}{1000}$$
$$\approx 0.693.$$

(1000 terms)

47. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3}$

By Theorem 9.15,

$$\left|R_N\right| \leq a_{N+1} = \frac{1}{(N+1)^3} < 0.001$$
$$\Rightarrow (N+1)^3 > 1000 \Rightarrow N + 1 > 10.$$

Use 10 terms.

49. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n^3 - 1}$

By Theorem 9.15,

$$\left|R_N\right| \le a_{N+1} = \frac{1}{2(N+1)^3 - 1} < 0.001.$$

This inequality is valid when $N = 7$. Use 7 terms.

51. $\sum_{n=1}^{\infty} \frac{(-1)^n}{2^n}$

$\sum_{n=1}^{\infty} \frac{1}{2^n}$ is a convergent geometric series.

Therefore, $\sum_{n=1}^{\infty} \frac{(-1)^n}{2^n}$ converges absolutely.

53. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$

$\frac{1}{n!} < \frac{1}{n^2}$ for $n \ge 4$

and $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is a convergent p-series.

So, $\sum_{n=1}^{\infty} \frac{1}{n!}$ converges, and

$\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$ converges absolutely.

55. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(n+3)^2}$

$\sum_{n=1}^{\infty} \frac{1}{(n+3)^2}$ converges by a limit comparison to the p-series $\sum_{n=1}^{\infty} \frac{1}{n^2}$. Therefore, $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(n+3)^2}$ converges absolutely.

57. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$

The given series converges by the Alternating Series Test, but does not converge absolutely because

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$$

is a divergent p-series. Therefore, the series converges conditionally.

59. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n^2}{(n+1)^2}$

$$\lim_{n\to\infty} \frac{n^2}{(n+1)^2} = 1$$

Therefore, the series diverges by the nth-Term Test.

61. $\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$

The series converges by the Alternating Series Test.

Let $f(x) = \frac{1}{x \ln x}$.

$$\int_2^{\infty} \frac{1}{x \ln x}\, dx = \Big[\ln(\ln x)\Big]_2^{\infty} = \infty$$

By the Integral Test, $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ diverges.

So, the series $\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$ converges conditionally.

63. $\sum_{n=2}^{\infty} \frac{(-1)^n n}{n^3 - 5}$

$\sum_{n=2}^{\infty} \frac{n}{n^3 - 5}$ converges by a limit comparison to the p-series $\sum_{n=2}^{\infty} \frac{1}{n^2}$. Therefore, the given series converges absolutely.

65. $$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}$$

$$\sum_{n=0}^{\infty} \frac{1}{(2n+1)!}$$

is convergent by comparison to the convergent geometric series

$$\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n$$

because

$$\frac{1}{(2n+1)!} < \frac{1}{2^n} \text{ for } n > 0.$$

Therefore, the given series converges absolutely.

67. $$\sum_{n=0}^{\infty} \frac{\cos n\pi}{n+1} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}$$

The given series converges by the Alternating Series Test, but

$$\sum_{n=0}^{\infty} \frac{|\cos n\pi|}{n+1} = \sum_{n=0}^{\infty} \frac{1}{n+1}$$

diverges by a limit comparison to the divergent harmonic series,

$$\sum_{n=1}^{\infty} \frac{1}{n}.$$

$\lim_{n\to\infty} \frac{|\cos n\pi|/(n+1)}{1/n} = 1$, therefore, the series converges conditionally.

69. $$\sum_{n=1}^{\infty} \frac{\cos n\pi}{n^2} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$$

$\sum_{n=1}^{\infty} \frac{1}{n^2}$ is a convergent p-series.

Therefore, the given series converges absolutely.