### 2.1 Relations and Functions

#### A: Relations

What is a relation? A _______________ of items from two sets:

A set of _______________ values and a set of _______________ values.

What does a relation look like?

---

**Example 1:** (pg 61)
The monthly average water temperature of the Gulf of Mexico in Key West, Florida varies during the year. In January, the average water temperature is 68°F, in February 70°F, in March, 75°F, and in April, 78°F. How can you represent this relation in four different ways?

---

<table>
<thead>
<tr>
<th>Ordered Pairs</th>
<th>Table of Values</th>
<th>Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x, y)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(−3, 4)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3, −1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4, −1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4, 3)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

---

**Mapping**
Domain and Range

What is domain?

The domain is the set of _______________ values, or _____________________________.

What is range?

The range is the set of ______________ values, or _________________________________.

Example 2: (pg 62)

What are the domain and range of this relation: \(\{(−3, 14), (0, 7), (2, 0), (9, −18), (23, −99)\}\)

B: Functions

What is a function?

A function is a special type of relation where each ______________ is paired with exactly one _________________. Another way to say this is that a function has ____________________________ !

Example 3: (pg 62)

Is the relation a function?

3a. \(\{(2, −3), (3, −1), (4, 3), (2, 3)\}\)

3b. \(\{(−7, 14), (9, −7), (14, 7), (7, 14)\}\)

Vertical Line Test

If your relation is represented as a graph, the easiest way to check if it is a function or not is by using the vertical line test.

What to do:

1. Imagine a vertical line passing over your graph from left to right.  
   (You can use your pencil to help you visualize this)

2. a. If the vertical line ALWAYS passes through ONLY ONE point on the graph at a time, the relation is a function.  
   b. If the vertical line EVER passes through MORE THAN ONE point on the graph at the same time, the relation is NOT a function.
Example 4: (pg 63)

Which graphs represent functions?

Function Notation

A function rule is an equation that represents an output value in terms of an input value.

For example: \( y = 3x + 2 \)

In function notation: \( f(x) = 3x + 2 \)

Example 5: (pg 63)

Evaluate the function for the given value of \( x \), then write the input \( x \) and output \( f(x) \) as an ordered pair.

5a: \( f(x) = -4x + 1 \) for \( x = -2 \)

5b: \( g(x) = x^2 - 3x \) for \( x = 1 \)

Example 6: Evaluate

Given: \( f(x) = x - 5 \) \( g(x) = 3x^2 \) \( h(x) = \frac{1}{2}x + 4 \)

6a: \( f(8) \) \hspace{2cm} 6b: \( h(-10) \) \hspace{2cm} 6c: \( g(0.55) \)

6d: \( 3f(-2) + 4g(2) - h(-6) \)
Writing and Evaluating a Function
To write a function you need to know which variable DEPENDS upon the other... what is the INPUT? the OUTPUT?

Example 6: (pg 64) You are buying bottles of a sports drink for a softball team. Each bottle costs $1.19.
    a) What function rule models the total cost of a purchase?

    b) Evaluate the function for 15 bottles.

Example 7: A telephone company charges a monthly service fee of $7.50 in addition to its rate of $0.15 per minute. Write a function for the cost per month. Evaluate for 230 minutes used.

2.3 Linear Functions and Slope-Intercept Form

A linear function is a relation whose ________________ is a _______________.

(Exception: An equation in the form ___________ is a vertical line, and is ________ a function!)

A: Slope

What is slope? Slope is the ratio of the vertical change (the rise) to the horizontal change (the run) between two points on a line.

How to find slope:

Key Concept: Slope

The slope of a nonvertical line through points \((x_1, y_1)\) and \((x_2, y_2)\) is the ratio of the vertical change to the corresponding horizontal change.

\[
slope = \frac{\text{vertical change (rise)}}{\text{horizontal change (run)}} = \frac{y_2 - y_1}{x_2 - x_1}, \text{ where } x_2 - x_1 \neq 0
\]
Example 1: (pg 75) Find the slope of the line that passes through the given points.

1a. (5, 4) and (8, 1)  1b. (2, 2) and (-2, -2)  1c. (9, -4) and (9, 3)

Four Types of Slope:

- **Positive Slope**: Line rises from left to right
- **Negative Slope**: Line falls from left to right
- **Zero Slope**: Horizontal line
- **Undefined Slope**: Vertical line

B: Slope Intercept Form

One way to write the equation of a linear function (a linear equation) is slope-intercept form.

Key Concept  Slope-Intercept Form

The slope-intercept form of an equation of a line is \( y = mx + b \), where \( m \) is the slope of the line and \((0, b)\) is the y-intercept.

Example 2: (pg 76) Write an equation for each line.

2a. \( m = \frac{1}{5} \) and y-intercept is (0, -3)

2b. \( m = 6 \) and y-intercept is (0, 5)

2c. 

2d.
Example 3: (pg 77) Write the equation in slope intercept form. What are the slope and $y$-intercept?

3a. $3x + 2y = 18$  
3b. $-7x - 5y = 35$

Helpful Hint: Solve for $y$!

C: Graphing a Linear Equation

Example 4: (pg 77) Graph each equation

4a. $y = -4x + 5$  
4b. $4x - 7y = 14$
4c. \( y + 5 = \frac{1}{2}x \)  

4d. \( y = 4 \)  

4e. \( x = -2 \)

2.4 More About Linear Equations

In order to graph an equation, you need to know either ______________________on the line,
or you need to know _______________ on the line and the ___________ of the line.

If you know this information, you can also use it to find the EQUATION of any line.

A: Point-Slope Form

Key Concept Point-Slope Form

The equation of a line in point-slope form through point \((x_1, y_1)\) with slope \(m\):

\[
y - y_1 = m(x - x_1)
\]

Example 1: (pg 82) Write the equation of the line

1a. slope = -3 through (7, -1)  
1b. slope = \(\frac{5}{6}\) through (22, 12)
Example 2: (pg 82) Write the equation of the line

2a. through (-5, 0) and (0, 7)  
2b. through (1, 4) and (-2, 7)

First! FIND SLOPE!

Example 3: There is no line that “Perfectly fits” the graph below. Write an equation for a line that approximates the data – a “good enough estimate” line.

B: Graphing Using Intercepts
A quick and easy way to find two points on a line so you can graph it is to find the x-intercept and the y-intercept.

To find the x-intercept: Set _____________, then solve for ___________.

To find the y-intercept: Set _____________, then solve for ___________.

Example 4:
Find the intercepts and then graph the line.
\[8x - 6y = -24\]
Many times you will be asked to graph or to find the equation of a line that is parallel or perpendicular to another line. In order to do this, you MUST ____________________ the slope of the given line and understand the ____________________ between parallel and perpendicular lines.

Examples of parallel slopes:

Example 6: (pg 85) What is the equation of each line in slope-intercept form?

6a. The line parallel to $4x + 2y = 7$ through (4, -2)  
6b. The line perpendicular to $2x - 3y = 3$ through (0,6)
2.5 Using Linear Models

Real world data usually doesn’t fit on a perfectly straight line.

However, we can ________________ real world data using a linear equation if the data appears to have a trend.

**A: Scatterplots**

*Take data.
*Plot as ordered pairs.
*Ask: Is there a correlation? (a trend/relationship)

The more the data “looks” like a line, the stronger the correlation, and the more accurately you can make predictions.

Example 1: (page 93)
The table shows the numbers of hours students spent online the day before a test and the scores on the test.

Make a scatterplot.
How would you describe the correlation?
B: Trend Lines

Once you have a scatterplot and have determined there is a correlation, you can draw in a trend line.

Example 2: (pg 94)
The table shows the median home prices in Florida.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Median Price ($)</td>
<td>23,100</td>
<td>40,100</td>
<td>58,100</td>
<td>57,600</td>
<td>89,300</td>
<td>98,500</td>
<td>105,500</td>
</tr>
</tbody>
</table>

What is the equation of a trend line that models the data? Use the equation to predict the median home price in 2020.

C: Linear Regression (Lines of Best Fit)

For this part of the notes, we will be using a TI 83 GRAPHING Calculator.

A graphing calculator (or scatterplot tool) can calculate a line of best fit through your data. This function is called “Linear Regression” and will tell you the slope (m), the y-intercept (b), and the correlation coefficient (r).

The closer the correlation coefficient (r) is to -1 or 1, the more accurate your equation and the better your predictions will be.

Examples:
Example: Use your graphing utility to find the equation of the line of best fit for the movie ticket data. Use your equation to predict the cost in 2025.

<table>
<thead>
<tr>
<th>Year</th>
<th>Average Cost of Movie Ticket</th>
</tr>
</thead>
<tbody>
<tr>
<td>1995</td>
<td>4.35</td>
</tr>
<tr>
<td>1997</td>
<td>4.59</td>
</tr>
<tr>
<td>1999</td>
<td>5.06</td>
</tr>
<tr>
<td>2001</td>
<td>5.65</td>
</tr>
<tr>
<td>2003</td>
<td>6.03</td>
</tr>
<tr>
<td>2005</td>
<td>6.41</td>
</tr>
<tr>
<td>2007</td>
<td>6.88</td>
</tr>
</tbody>
</table>

Example: The table shows the population and the number of drivers in several states. Find a linear model (a line of best fit) for the data. If Michigan had a population about 10 million during this year, about how many drivers were in Michigan?

<table>
<thead>
<tr>
<th>State</th>
<th>Population (millions)</th>
<th>Number of Drivers (millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arkansas</td>
<td>2.7</td>
<td>1.9</td>
</tr>
<tr>
<td>Illinois</td>
<td>12.4</td>
<td>7.7</td>
</tr>
<tr>
<td>Kansas</td>
<td>2.7</td>
<td>1.8</td>
</tr>
<tr>
<td>Massachusetts</td>
<td>6.4</td>
<td>4.4</td>
</tr>
<tr>
<td>Pennsylvania</td>
<td>12.3</td>
<td>8.3</td>
</tr>
<tr>
<td>Texas</td>
<td>20.9</td>
<td>12.8</td>
</tr>
</tbody>
</table>
2.6 Families of Functions

A: Parent Functions and Transformations

Certain functions can be grouped together into “families.”

Each family has a parent function which is the ______________________ of the set of functions.

Every other function in the family is called a ______________________ of the parent function.

EXAMPLE: The Linear Functions

The most simple linear function is \( y = x \).

PARENT: \( y = x \)

In a different color: Graph the transformation \( y = x - 4 \)

What do you notice?

B: Translations: Vertical and Horizontal

A translation occurs when the graph is ______________________ vertically or horizontally.

PARENT: \( y = f(x) \)

Vertical Translations:

Shifted up \( K \) units: \( y = f(x) + k \)

Shifted down \( K \) units: \( y = f(x) - k \)

Horizontal Translations:

Shifted left \( h \) units: \( y = f(x + h) \)

Shifted right \( h \) units: \( y = f(x - h) \)
Example:

a) Graph the parent graph $y = x^2$

b) Sketch the graph of $y = x^2 + 3$

c) Sketch the graph of $y = (x - 2)^2$

d) Sketch the graph of $y = (x + 5)^2$

Example: Given the parent graph, $y = \sqrt{x}$, write the equation of the transformation that has been translated:

a) 3.5 units down

b) 2.5 units left

c) 6 units up

d) 7 units right

C: Reflections

A reflection __________ the graph across the x or y axis.

Reflection across y-axis:

$y = f(-x)$

y values stay same
x values change sign +/=

Reflection across x-axis:

$y = -f(x)$

x values stay same
y values change sign +/-
Examples of reflections:

Example:

\[
\begin{align*}
y &= x^2 \\
y &= -x^2 \\
y &= 2x^3 - 4 \\
y &= -2x^3 - 4
\end{align*}
\]

D: Stretching and Compressing

A vertical stretch multiplies all y-values of a function by a ______________.

A vertical compression reduces all y-values of a function by a ______________.

Vertical Stretch/Compression: \[ y = a \cdot f(x) \]

Example:
Describe the transformations change the graph of \( f(x) \) to \( g(x) \).

a) \( g(x) = 4f(x) \)  
   b) \( g(x) = \frac{1}{3}f(x) \)

Examples of stretches and compressions:

\[
\begin{align*}
y &= x^2 \\
y &= 10x^2 \\
y &= \frac{1}{4}x^2 \\
y &= -8x^2
\end{align*}
\]
E: Putting it all together!

Example: Describe the transformations that take place on the parent graph $y = x^2$.

a) $y = 3(x + 2)^2 - 8$

b) $y = -(x - 5)^2 + 2$

c) $y = \frac{1}{4}(x + 1)^2 + 3.5$

d) $y = -2x^2 + 5$

e) $y = (x + 4)^2$

f) $y = -0.35(x - 3)^2$
2.7 Absolute Value Functions and Graphs

A: Absolute Value Graphs

Recall that the absolute value of \( x \) is the distance of \( x \) from zero on a number line.

Use a t-chart to graph \( y = |x| \)

This is the simplest absolute value equation... so this is the absolute value PARENT GRAPH.

IMPORTANT PARTS OF AN ABSOLUTE VALUE GRAPH
• VERTEX
• AXIS of SYMMETRY

SHAPE OF AN ABSOLUTE VALUE GRAPH

If we apply transformations to an absolute value graph we get an equation that looks like:

Key Concept General Form of the Absolute Value Function

\[ y = a|x - h| + k \]

The stretch or compression factor is \( |a| \), the vertex is located at \( (h, k) \), and the axis of symmetry is the line \( x = h \).

Example 2: (pg 109)

Multiple Choice Which of the following is the graph of \( y = |x + 2| + 3 \)?
Example: Use a table of values, or transformations to graph each absolute value equation. State the vertex and the axis of symmetry for each.

a. \( y = |x| + 3 \)

b. \( y = |x - 5| \)

c. \( y = -2|x| \)

d. \( y = 3|x + 2| - 5 \)

e. \( y = \frac{1}{2}|x| + 4 \)

f. \( y = |2x - 8| \)
Concept Byte: Piecewise Functions

A piecewise function is a function with ______________ rules for ______________ parts of the domain.

The graph of a piecewise function appears to be ____________________________.

Example 1: Look at the graph to the right

Example 2: (page 90)

Graph $f(x) = \begin{cases} -2x + 3, & \text{if } x < 2 \\ x - 1, & \text{if } x \geq 2 \end{cases}$

Example 3:

Graph $f(x) = \begin{cases} x + 2, & \text{if } x \leq -1 \\ 3x, & \text{if } x > -1 \end{cases}$. 
Example 4:

Graph \( f(x) = \begin{cases} 
  x + 2, & \text{for } x < -5 \\
  4, & \text{for } -5 \leq x < 1 \\
  -3x, & \text{for } x \geq 1 
\end{cases} \)

**Step Functions**

A Step Function is a special kind of piecewise function where every number in an interval is paired with a single value. The graph of a step function can look like the steps of a staircase.

Example 5:

In 2008, first-class letter postage was $.42 for up to one ounce and $.17 for each additional ounce up to 3.5 oz. Graph this postage function.