5.1: Rate of Change and Slope

Rate of Change – shows relationship between changing quantities.

\[
\text{rate of change} = \frac{\text{Change in dependant variable}}{\text{Change in independant variable}}
\]

On a graph, when we compare rise and run, we are talking about steepness of a line (slope).
You can use and 2 points on a line to find slope.

\[
\text{Slope (m)} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{rise}}{\text{run}}
\]

Example: Find the slope of the line.

There is also a formula to use to find slope. Given two points \((x_1, y_1)\) and \((x_2, y_2)\)

\[
\text{Slope (m)} = \frac{y_2 - y_1}{x_2 - x_1} ; \text{ where } (x_1, y_1) \text{ and } (x_2, y_2) \text{ are points in the plane}
\]

You can either use the formula, or you can plot points and count.
REDUCE as much as possible, but leave in fraction form.

Example: Find the slope of a line that passes through the given points

1) \((-1,6)\) and \((5, 8)\)  2) Passing through \((-1, -2)\) and \((-4, 1)\)  3) Passing through \((-4, -2)\) and \((-8, -3)\)

4) Passing through \((4, 5)\) and \((4, 7)\)  5) Passing Through \((1, 2)\) and \((-1, 2)\)
Models of Slope

<table>
<thead>
<tr>
<th>Positive</th>
<th>Negative</th>
<th>Zero/Horizontal</th>
<th>Undefined/Vertical/No Slope</th>
</tr>
</thead>
</table>

**Example:** Find the value of $s$ so that the line through $(s, 3)$ and $(7, 4)$ has a slope of $3$.

**Example:** Find the value of $r$ so that the line through $(r, 6)$ and $(10, -3)$ has a slope of $\frac{-3}{2}$.
Chapter 5 – Linear Functions

Name___________________

5.3: Slope Intercept Form

Slope Intercept Form:

\[ y = mx + b \]

\( m = \text{slope of the line, } b = \text{y-intercept} \)

So to put an equation in slope intercept form you must find two things

- a slope (m)
- the y-intercept (b)

Y-intercept:
the \( y \) – coordinate of the the point where the graph crosses the \( y \) axis.

\( (The \ value \ of \ x \ is \ always \ 0 \ at \ the \ y\text{-intercept}) \)

Example: What are the slope and y-intercept of the equation \( y = 5x - 2 \)?

Example: Write the equation of a line whose slope is \( -\frac{4}{5} \) and whose y-intercept is 7 in slope-intercept form.

Example: Write the equations of the lines represented on the following graphs
What happens when you don’t know either slope or the y-intercept?

1) What do you need first?

2) Ordered pairs represent what 2 variables in the equation \( y = mx + b \)?

Steps: (When you don’t know slope or the y-intercept.)

1) Find the slope using the given points. (Plug in for “m”)
2) Choose either point and plug in for \( x \) and \( y \).
3) Solve for “b”
4) Write the final equation using the “m” and the “b” that you found.

Example: Write an equation of the line passing through \((2,1)\) and \((5,-8)\) in slope-intercept form.

Example: Write an equation of the line passing through \((3,-2)\) and \((1,-3)\) in slope-intercept form.

Graphing Lines Using Slope and Y-Intercept

1) Get to slope-intercept form by solving for \( y \)
2) State what the slope is and the y-intercept.
3) Plot the y-intercept
4) From the y-intercept, use the slope by doing rise over run to get the next point.
5) Draw the line

Get the following equations to slope-intercept form.

1) \( 5x + 2y = 8 \)
2) \( 4x - 3y = -9 \)
Chapter 5 – Linear Functions

Example: Graph the lines:

1) \( y = 2x - 1 \)

2) \( y = \frac{-2}{3}x + 1 \)

3) \( 5x - 3y = 6 \)

Example: Story Problem:
A plumber charges a fee of $65 plus $35 per hour. Write an equation to model the total cost \( y \) of a repair that takes \( x \) hours. Then, make a graph that models this information.
Example: Bob measured the height of a flower in meters for three consecutive days. His results are shown in the table below.

<table>
<thead>
<tr>
<th>Day</th>
<th>Height</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.3</td>
</tr>
<tr>
<td>2</td>
<td>2.6</td>
</tr>
<tr>
<td>3</td>
<td>3.9</td>
</tr>
</tbody>
</table>

Based on the table, what is the equation that models the height(h) of the flower relative to the number of days(d)?
5.4: Point Slope Form

**Point Slope Form**

\[ y - y_1 = m(x - x_1) \]

- \( m = \text{slope} \)
- \((x_1, y_1) = \text{point the line passes through}\)

**What two things do you need based on the name?**

**Steps for writing in Point-Slope Form:**
1) Must know the slope (Find it out if given two points)
2) Must know one point the line passes through (Label this as your \( x_1 \) and \( y_1 \))
3) Plug into the equation

**Example:** Write the following in point-slope form for the line passing through the given point and having the given slope.

(1, 5); \( m = -2 \)  
(4,2) with slope \( \frac{1}{2} \)  
(-2, -3); \( m = \frac{1}{2} \)

**Graphing using Point-Slope Form**

*Ask yourself which one is easier?*

1. **Convert to** \( y = mx + b \)
2. **Plot the** \( y \)-intercept and use the slope
3. **State the** slope and the given point
4. **Plot the** given point and use the slope
Chapter 5 – Linear Functions

**Example:** What is the graph of \( y + 7 = -\frac{4}{5}(x - 4) \)?

**Example:** Write the point-slope form of an equation for a horizontal line passing through (6, -2).

*Hint: What is the slope of a horizontal line?*

**Example:** Write the point-slope form of a line passing through (2, 3) and (-1, 6)

**Example:** Write the equation of the line passing through (-2, 4) and (4, 8) in slope-intercept form.

**Example:** Write the equation of a line passing through the two points (-3, -1) and (6, -4) in slope-intercept form.
5.5: Standard Form

Ax + By = C

Mainly used in Chapter 6
Main Use is for Quick Graphing or Finding Intercepts

Quick Graphs:
1) Plug zero in for “x” and you find the y-intercept. (Write as (0 , intercept) )
2) Plug zero in for “y” and you find the x-intercept. (Write as (intercept , 0) )
3) Plot the two values and graph the line.

Examples: Determine what the x-intercept and the y-intercept are for the following equations.

1) 4x + 3y = 12  
2) 2x – 6y = -18  
3) 4x – 5y = 16

Examples: What is the graph of the following equations?

1) 2x + 3y = 12  
2) 4x – 5y = -20
Story Problems:
A person walks into a store and wants to buy pairs of jeans and t-shirts. Each pair of jean costs $35 and each t-shirt costs $7. Write an equation to represent the situation if the person spends a total of $140.

Why can’t we solve the equation that we wrote?

Example: Given the equation $ax + 3y = 12$ and the graph below, Find the value of ‘a’.

![Graph with points (0, 4) and (6, 0)]
5.6: Parallel and Perpendicular Lines

Parallel Lines:
1) Lines that never ____________________________
2) Lines that have the same ____________________

Writing the Equation of Parallel Lines Through a Given Point

Steps:
1) Find the slope of the given line (Use only the slope from the given equation, label as “m”)
2) Use that slope and the given point in point-slope form (Label the point as \((x_1, y_1)\))
3) Write the final equation in the specific form

Example: Write the slope-intercept form of an equation for the line passing through \((4, -2)\) and is parallel to the graph of \(y = \frac{1}{2}x - 7\).

Example: Write the slope-intercept form of an equation parallel to the line that passes through the points \((1, 5)\) and \((-4, 15)\).
Chapter 5 – Linear Functions

Name___________________

Perpendicular Lines:

1) Lines that ________________ and form ________________

2) Slopes are opposite reciprocals

Opposite Reciprocals $-\frac{2}{3}$ and $-\frac{3}{2}$

Steps for Writing the Equation of a Line Perpendicular to a Given Line:

1) Identify slope of the given line (Get to $y=mx + b$)
2) Take the opposite reciprocal (Flip the slope and change the sign)
3) Use the new slope and point to write the new equation by starting in point-slope form.

Example: Write the slope-intercept form of an equation passing through $(7, -1)$ and perpendicular to $y = \frac{7}{2}x - 5$

Example: Write the point-slope form of a line perpendicular to $2y + 5x = 2$ and passing through $(0, 6)$

Determining Parallel or Perpendicular

Convert to Slope-Intercept Form

Look at the slope

Same Slope and Different Y-Intercepts – parallel
Same Slope and Same Y-Intercepts – Same Line
Slopes are Opposite Reciprocals – perpendicular
Slope are Neither – neither

Examples: Determine if the following equation are parallel lines, perpendicular or neither.

1) $y = \frac{-2}{3}x + 2$
   $y = \frac{-2}{3}x - 2$

2) $3x - 5y = 10$
   $10x + 6y = 24$