Density Curves and Normal Distributions

Suppose we looked at an exam given to a large population of students. The histogram of this data appears like the graph to the left below. However, rather than show how many data values are in each bin, we now look at the bin height in terms of percentage. This histogram is symmetric and the tails fall off from a single peak in the center. There are no obvious outliers. We now draw a smooth curve through the tops of the bins to get a better picture of the overall pattern of the data (right graph). This curve is an approximate mathematical model for the distribution. We will find that it is easier to work with this model than the actual histogram. We call this curve a **density curve**. Since the height of the histogram is in terms of percent, it should be obvious that the total area under the histogram should be exactly 1 (as 100% of the scores are shown).

![Histogram and Smooth Curve](image)

**Definition of a Density Curve:**

- is always above the $x$-axis
- has area of exactly 1 underneath it

In the graph to the right, a density curve is shown. It appears to have data values from 0 to 10 and is skewed right. The shaded area under this density curve between 2 and 4 is .3109. That is saying that 31.09% of the data lies between 2 and 4.

The median of a density curve is the equals-area point, the point that divides the total area under the curve in half. The mean of a density curve is the balance point at which the curve would balance if made of solid material and place on a fulcrum.

In a symmetric distribution, the mean and median are the same. In a skewed distribution, the mean is pulled away from the median in the direction of the skew.
Example 1) To the right is a density curve for distribution of values between 0 and 1. It is called a uniform distribution (that is all values are equally likely). Use geometry to determine the answers to these questions.

a. What percentage of the data lies above 0.7?
b. What percentage of the data lies below 0.4?
c. What percentage of the data lies between .3 and .9?
d. What percentage of the data is exactly .8?
e. What is the median of the data?
f. What is the mean of the data?

Example 2) To the right is a density curve for distribution of values between 0 and 2. Use geometry to determine the answers to these questions.

a. What percentage of the data lies above 1?
b. What percentage of the data lies below 0.5?
c. What percentage of the data lies between .2 and .6?
e. What is the median of the data?
f. Where in relation to the median is the mean of the data?

Example 3) To the right is a density curve for distribution of values between 0 and .8. Use geometry to determine the answers to these questions.

a. What percentage of the data lies above .4?
b. What percentage of the data lies above .6?
c. What percentage of the data lies between .2 and .6?
e. Draw the approximate location of the median on the diagram.
f. Is the data skewed left or right?
g. Where in relation to the median is the mean of the data?

Example 4) A density curve consists of two lines. The first line goes from the point (0, .8) to (.6, 0). The second line goes from the point (.6, 0) to (1, a).

a. Find the value of a and draw the density curve to the right.
b. What percentage of the data lies below .6?
c. What percentage of the data lies below .9?
d. What percentage of the data lies above .5?
e. What percentage of data lies above .1?
f. What is the median of the data?
Normal Distributions

One of the most important distributions in statistics is called the normal distribution. If a histogram is “smoothed out”, many times its density curve will appear symmetric, single-peaked and bell-shaped. These are called normal curves or sometimes called the bell-curve. To the right is a picture of a normal curve. Note how most of the data is in the center and it tails off symmetrically to the sides with little amount of data at the far left and right.

The reason these curves are so important in statistics is that so much of real life statistics appears “normal.” Here are some common examples:

• Weights: most people are average weight with fewer people really skinny and fat.

• Heights: most people are average height with fewer people really short and really tall

• Most data about the human body is normal like fingernail length, hair length, how long people live, …

• Prices: go to many markets and price a 2-liter bottle of Pepsi. The data will appear normal with most prices about average, and fewer much cheaper and fewer much more expensive

• Wages: Most people make an average amount of money while fewer make little money and fewer make a lot of money.

• Time it takes to get to work: Most days it might take 30 minutes to drive to work while fewer days it might take 20 minutes and fewer days it might take 40.

• Grades: Collect GPAs for a class of students and many students will have an average GPA with fewer with low GPA’s and fewer with high GPA’s.

• Time before a class students arrive. Most students arrive at a class 3 minutes before the class begins fewer get their earlier and fewer get there after the class begins.

Not all distributions are normal. For instance, although the time before a class students arrive is normal as seen above, the time students leave a class is not normal. At the end of the period, most students leave in a big clump.

The normal distribution follows an important rule called the 68-95-99.7% rule. It states that in any normal distribution,

• 68% of the data lies within one standard deviation of the mean

• 95% of the data lies within two standard deviations of the mean

• 99.7% (or just about all) of the data lies within three standard deviations of the mean.

The more “normal” a distribution is, these relationships become closer to being perfectly true. No distribution is perfectly normal and therefore, these relationships are approximations in most real-life settings.
Example 1) The distribution of heights of adult American men is approximately normal with mean 69 inches and standard deviation 2.5 inches. On the normal curve below, mark the values for the mean and 1, 2, and 3 standard deviations above and below the mean. Then answer the questions.

a) What percent of men are shorter than 64 inches?

b) What percent of men are taller than 71.5 inches?

c) What percent of men are taller then 66.5 inches?

d) Between what heights do the middle 95% of men fall?

e) A height of 74 inches corresponds to what percentile of American male heights?

Example 2) The length of human pregnancies from conception to birth varies on an approximate normal distribution with mean 266 days and standard deviation 16 days. On the normal curve below, mark the values for the mean and 1, 2, and 3 standard deviations above and below the mean. Then answer the questions.

a) How long are the longest 16% of pregnancies?

b) How short are the shortest 16% of pregnancies?

c) Between how many days does the middle 95% of all pregnancies fall?

d) Between how many days does the middle 99.7% of all pregnancies fall?

e) A pregnancy of 234 days corresponds to what percentile of pregnancies?

z - scores

Suppose 4 students took tests on the same subject given by different teachers. Following are the number of points in the test and the score of the student. Rate the students from best grade to worst grade.

<table>
<thead>
<tr>
<th>Student</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Points</td>
<td>100</td>
<td>45</td>
<td>18</td>
<td>75</td>
</tr>
<tr>
<td>Student</td>
<td>83</td>
<td>37</td>
<td>15</td>
<td>63</td>
</tr>
</tbody>
</table>

Best score _____ 2nd best score _____ 3rd best score _____ Worst score _____

In order to do this, you realized that you couldn’t look at just the score because each exam was based on a different number of points. You needed to find a common ground to compare them. This is called standardizing. That was percentage. The word percentage means per one hundred. Finding a percentage allows you to determine the highest by deciding which number is the largest.
In a previous example, we found that not all classes that share the same mean in an exam have the same grades. They may differ greatly in terms of spread.

**z-score definition:** A z-score allows us to standardize a score $x$ if we know that the distribution of the scores is normal (remember – a bell-shaped curve) and we know the distributions mean $\bar{x}$ and standard deviation $s$. The z-score typically runs between -3 and 3. It represents the number of standard deviations a score is above or below the mean. If the z-score is positive, the score is above the mean and if the z-score is negative, the score is below the mean.

Example 3) If four students took exams and received the following scores when we know the mean of the exam and its standard deviation, determine which student had the higher score *relative to his class.* Assume the distribution of each exam is normal. Use your calculator to determine the z-score.

<table>
<thead>
<tr>
<th>Student</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student Score - $x$</td>
<td>80</td>
<td>28</td>
<td>13</td>
<td>98</td>
</tr>
<tr>
<td>Mean of test - $\bar{x}$</td>
<td>75</td>
<td>25</td>
<td>12</td>
<td>92.4</td>
</tr>
<tr>
<td>Standard deviation of test - $s$</td>
<td>10</td>
<td>4</td>
<td>1</td>
<td>6.3</td>
</tr>
</tbody>
</table>

**z-score**

Best score ______ 2nd best score ______ 3rd best score ______ Worst score ______

Remember … this only works when the distribution is normal (bell curve following the 68-95-99.7 rule)

Because so many distributions are normal, this technique is used a great deal.

Example 4) 4 runners run the New York marathon. We wish to determine which runner had a faster time relative to his age group. Assume normality. Complete the chart.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Time – $x$</td>
<td>2 hours, 45 min</td>
<td>2 hours, 59 min</td>
<td>3 hours, 39 min</td>
<td>3 hours, 1 min</td>
</tr>
<tr>
<td>Mean time - $\bar{x}$</td>
<td>3 hours, 2 min</td>
<td>3 hours, 23 min</td>
<td>4 hours, 27 min</td>
<td>3 hours, 41 min</td>
</tr>
<tr>
<td>Standard deviation - $s$</td>
<td>16 min</td>
<td>19 min</td>
<td>28 min</td>
<td>22 min</td>
</tr>
<tr>
<td>z-score</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Best time ______ 2nd best time ______ 3rd best time ______ Worst time ______
Example 5) Over a ten year period, American League baseball players and National League players had the following averages and standard deviations. Averages over that long a period of time would be essentially normal.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>American League</td>
<td>.263</td>
<td>.202</td>
</tr>
<tr>
<td>National League</td>
<td>.254</td>
<td>.214</td>
</tr>
</tbody>
</table>

a. Jim Thome played in both leagues and had a batting average of .277 in the American League and .273 in the National League. Show work to determine what league his batting average was higher compared to the rest of the league.

b. Aaron Rowand played in both leagues and had a batting average of .250 in the American League and .241 in the National League. Show work to determine what league his batting average was higher compared to the rest of the league.

c. If a player plays in both leagues and bats .300 in the National League, show appropriate work to determine what average he would have to have in the American League so that his average would be the same compared to the rest of the league.

Standard Normal Calculations

If we know a distribution is normal and know its mean and standard deviation, we can easily answer questions as to what percentage of data lies between two values using our calculator. In the preceding section, we used percentages like 68% and 95% but now we can be more specific.

Example 6) The heights of American men is approximately normal with mean of 69 inches and standard deviation of 2.5 inches.

a) what percentage of men are over 6 feet (72 inches) tall?

Step 1) Draw a picture:

Step 2: Calculate the z-score. 
\[ z = \frac{72 - 69}{2.5} = 1.2 \]

This is saying that 72 inches is 1.2 standard deviations above the mean.

Step 3: Use the table of standard normal probabilities in your book. Look up your z-value and find the area under the standard normal curve that corresponds to the picture that the table gives.

Step 4: If the picture you drew above is identical to the picture in the table, you have your answer. If not, you will have to subtract your answer from 0.5. The answer to the problem is 11.51%

b) what percentage of men are under 5 feet, 4 inches tall?

\[ z = \frac{64 - 69}{2.5} = -2 \] , percentage is 2.28%

c) what percentage of men are between 5 feet, 5 inches and 6 feet tall?

since your values are both below and above the mean, you need two z scores

\[ z = \frac{72 - 69}{2.5} = 1.2 \] , Area (unshaded) = .1151

\[ z = \frac{65 - 69}{2.5} = -1.6 \] , Area (unshaded) = .0548

Area (shaded) = 1 - .1151 - .0548 = 83.01%
Using the table is cumbersome (although you need to know how to use it). This is all built into your calculator.

The two statements we want to work with **normalcdf** and **invNorm**. They are located in the DISTR(distribution) menu which is ![2nd VARS](image). When you press ![2nd VARS](image), you will get this screen. **normalcdf** is number 2 (don’t mistake it with **normalpdf** which is number 1). **invNorm** is number 3. The best way to illustrate this is to use an example.

Example 7) The heights of American men is approximately normal with mean of 69 inches and standard deviation of 2.5 inches.

a) what percentage of men are between 5 feet (60 inches) and 6 feet (72 inches) tall?

Solution: You know the height and you want the percent so you will use the **normalcdf** statement. The **normalcdf** command is in this syntax: 
\[ \text{Normalcdf}(\text{left, right, mean, standard deviation}) \]. In this case, we know all that information. The screen on the right shows you the answer which is 88.5%

b) what percentage of men are over 6 feet tall?

Solution: We know the left – 72 inches, but the right goes out to infinity. Simply enter some large number for the right like 100. The answer is 11.5%

c) what percentage of men are under 5 feet, 4 inches tall?

Solution: We know the right, 64 inches and the left is 0. The answer is 2.3%

What percentage of men are:

d) over 5 feet tall  
e) under 6’2”  
f) between 5’8” and 6’1”

g) How tall must a man be to be in the 90\(^{th}\) percentile and 99\(^{th}\) percentile of height for American men

Solution: In this case, you know the percent and you want the height.

Without the calculator, you have to use the table backwards. You know that some height \(x\) represents the 90\(^{th}\) percentile meaning that 10% of the heights are above \(x\). So you look in the middle of your table and find the \(z\)-score associate with .10. The closest value of \(z\) is 1.28. So, by formula:

\[
z = 1.28 = \frac{x - 69}{2.5} \text{ or } x - 69 = 3.2 \text{ or } x = 72.2 \text{ inches}
\]

For the 99\(^{th}\) percentile, only 1% are above \(x\). The \(z\)-score associated with .01 is 2.33 so:

\[
z = 2.33 = \frac{x - 69}{2.5} \text{ or } x - 69 = 5.83 \text{ or } x = 74.83 \text{ inches}
\]
Again, this is easier to perform on the calculator. Use the \texttt{invNorm} statement which is in this syntax:

\begin{center}
\texttt{InvNorm(percent, mean, standard deviation)}.
\end{center}

In this case we know all that information. The screen on the right shows you the answer which is 72.2 inches.

h) How tall must a man be to be in the top 1% of height for American men?
Solution: The top 1% means being the 99th percentile. The answer is 74.8 inches.

\textbf{Remember: when you want a percentage, you use \texttt{Normcdf. When you have a percentage, you use InvNorm.}}

Example 8) Joan found that her average telephone call last month was 9.6 minutes with a standard deviation of 2.4 minutes. Her telephone call usage was roughly normal. What percentage of her calls

\begin{enumerate}
\item[\textbf{a.}] were less than 5 minutes?
\item[\textbf{b.}] were less than 10 minutes?
\item[\textbf{c.}] were more than 15 minutes?
\item[\textbf{d.}] were between 4 and 8 minutes?
\end{enumerate}

Find the telephone call length that represents:

\begin{enumerate}
\item[\textbf{e.}] the 90th percentile
\item[\textbf{f.}] the 98th percentile
\item[\textbf{g.}] the top 0.5 percent
\item[\textbf{h.}] the bottom 1%
\end{enumerate}

Example 9) The videos at Hollywood Video are roughly normal in length with an average length of 2 hours 3 minutes and a standard deviation of 17.25 minutes. If I pick a video at random, what is the probability that it will be

\begin{enumerate}
\item[\textbf{a.}] longer than 2.5 hours
\item[\textbf{b.}] shorter than 1 hour, 45 minutes
\item[\textbf{c.}] longer than an hour and a half
\item[\textbf{d.}] between 2 and 2.25 hours
\end{enumerate}

Find the video length that represents:

\begin{enumerate}
\item[\textbf{e.}] the 95th percentile
\item[\textbf{f.}] the 99.9th percentile
\item[\textbf{g.}] the bottom 3%
\item[\textbf{h.}] the middle 15%
\end{enumerate}
Density Curves and Normal Distributions - Homework

1) To the right is a density curve for distribution of values between 0 and 1.3. Use geometry to determine the answers to these questions.

   a. What percentage of the data lies below .6?
   b. What percentage of the data lies above .5?
   c. What percentage of the data lies between .3 and .9?
   d. What percentage of the data is exactly .8?
   e. What is the median of the data?
   f. Draw the approximate position of the mean on the drawing

2) To the right is a curve for distribution of values between 0 and .5. Use geometry to determine the answers to these questions.

   a. Show that it is a density curve.
   b. What percentage of the data lies above .3?
   c. What percentage of the data lies below .2?
   d. What percentage of the data is below .1 or above .4?
   e. What is the median of the data?
   f. Draw the approximate position of the mean on the drawing

3) A density curve consists of two lines. The first line goes from the point (1, 0) to (3, .4). The second line goes from the point (3, .4) to (a, 0).

   a. Find the value of a and draw the density curve to the right.
   b. What percentage of the data lies below 2?
   c. What percentage of the data lies above 4.5?
   d. What percentage of the data lines between 2 and 4.5?
   e. If the median is at 3.26, where is the mean in relation?

4) If 16 coins are flipped, any number of heads between 0 and 16 could come out. Below is a table of every possibility. For instance, there are 560 ways that exactly 3 heads could come out. The mean of the distribution is exactly 8 and the standard deviation of the data is exactly 2.

<table>
<thead>
<tr>
<th>Heads</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>1</td>
<td>16</td>
<td>120</td>
<td>560</td>
<td>1820</td>
<td>4368</td>
<td>8008</td>
<td>11440</td>
<td>12,870</td>
<td>11,440</td>
<td>8,080</td>
<td>4,680</td>
<td>1,820</td>
<td>560</td>
<td>120</td>
<td>16</td>
<td>1</td>
</tr>
</tbody>
</table>

   a) Create a histogram for the data.
   b) Make a statement about the “normality” of the data.
5) Purchases at a supermarket on a busy Saturday afternoon fall into a roughly normal distribution with mean $55 and standard deviation $18. On the normal curve below, mark the values for the mean and 1, 2, and 3 standard deviations above and below the mean. Then answer the questions.

a) How much do the middle 68% of customers purchase?

b) How much do the middle 95% of customers purchase?

c) What percentage of people spends more than $73?

d) What percentage of people spends less than $19?

e) A purchase of $91 corresponds to what percentile of spending?

6) Since the year 1800, records have been kept on the snowfall in the Philadelphia area which follow roughly a normal distribution. The average snowfall has been 59.9 inches with a standard deviation of 18.6 inches. On the normal curve below, mark the values for the mean and 1, 2, and 3 standard deviations above and below the mean. Then answer the questions.

a) What percentage of winters get less than 78.5 inches?

b) What percentage of winters get more than 22.7 inches?

c) How much snowfall do the middle 95% of winters get?

d) How much snowfall do the middle 99.7% of winters get?

e) A snowfall of 41.3 inches corresponds to what percentile of snowfall?

7) The rate at which people read is approximately normal. Suppose that the average time to read a novel is 7 hours with a standard deviation of 1.5 hours. On the normal curve below, mark the values for the mean and 1, 2, and 3 standard deviations above and below the mean. Then answer the questions.

a) What percent take less than 8.5 hours to read the book?

b) What percent take more than 4 hours to read the book?

c) What percent of people take between 4 and 5.5 hours?

d) What percent of people take between 2.5 and 4 hours?

e) A time of 8.5 hours corresponds to what percentile of American male readers? (assume faster is better).
8) A consumer groups tries to determine the most expensive items in a convenience store. Four items are purchased in the convenience store and compared to the average price in stores in the area. Assume that the price distribution is normal. Use your calculator to determine the z-score for each.

<table>
<thead>
<tr>
<th>Item</th>
<th>2 liter Pepsi</th>
<th>12 oz potato chips</th>
<th>Loaf of bread</th>
<th>Pizza</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item price - x</td>
<td>2.19</td>
<td>3.79</td>
<td>1.89</td>
<td>13.99</td>
</tr>
<tr>
<td>Item mean price - ( \bar{x} )</td>
<td>1.69</td>
<td>3.29</td>
<td>1.49</td>
<td>11.58</td>
</tr>
<tr>
<td>Stand. Deviation price. - s</td>
<td>.22</td>
<td>.19</td>
<td>.13</td>
<td>1.17</td>
</tr>
<tr>
<td>z-score</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Most expensive _____ 2\(^{nd}\) most expensive _____ 3\(^{rd}\) most expensive _____ cheapest _____

9) In the year 2000, there were 125 major new automobile types available. If you look at the miles per gallon (MPG) for these vehicles, the distributions are roughly normal. The means and standard deviations are given in the following chart:

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>City MPG</td>
<td>22.37</td>
<td>4.77</td>
</tr>
<tr>
<td>Highway MPG</td>
<td>29.09</td>
<td>5.46</td>
</tr>
</tbody>
</table>

a. The Lincoln Continental had a rating of 17 mpg city and 24 mpg highway. Which rating, city or highway, is higher compared to the other cars? Show work.

b. The Saturn SL had a rating of 39 mpg city and 43 mpg highway. Which rating, city or highway, is higher compared to the other cars? Show work.

c. The Mazda 626 had a city rating of 26 mpg. What highway rating would it have so that the city and highway ratings were the same compared to other cars?

10) Two neighboring school districts, Newton and Center Valley compare salaries for its teachers. Assume normality for distribution of both salaries.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Harrison</td>
<td>$51,033</td>
<td>$8,654</td>
</tr>
<tr>
<td>Center Valley</td>
<td>$52,176</td>
<td>$9,299</td>
</tr>
</tbody>
</table>

a. Mr. and Mrs. Newton are married. Mr. Newton works at Harrison and makes $57,000 while Mrs. Newton works at Center Valley and makes $58,250. Who is higher paid within their school district? Show work.

b. Mr. and Mrs. Sun are married. Mr. Sun works at Harrison and makes $47,000 while Mrs. Sun works at Center Valley and makes $48,000. Who is higher paid within their school district? Show work.
11) In a city parking lot, the average age of a car is 4.65 years with a standard deviation of 1.24 years. Assuming the distribution of cars is normal. If a car is chosen at random, find the percentage of cars that are

a. older than 5 years  

b. newer than 2 years

c. less than 1 year old  

d. between 3 and 6 years old

Find the car age that represents:

e. the 85th percentile  

f. the 99.5 percentile

g. the oldest 2%  

h. the youngest 10%

12) Suppose the systolic blood pressure of an adult male is normally distributed with a mean of 138 mm of mercury and standard deviation of 10. If an adult male is picked at random, find the percentage of men whose systolic blood pressure will be

a. greater than 160 mm  

b. between 120 and 135 mm

c. less than 110 mm  

d. between 130 and 140 mm

Find the systolic blood pressure for men that represents:

e. the 99th percentile  

f. the 99.5th percentile

g. the top 0.01 percent  

h. the bottom 0.5 percent

13) The shelf life of milk (properly refrigerated) is normally distributed with a mean of 9 days and standard deviation of 2.8 days. If a gallon of milk is randomly selected what percentage of milk

a. will last more than 2 weeks  

b. will go bad within a week

c. will last between 1 and 2 weeks  

d. will last between 8 and 10 days

Find the milk shelf length that represents

e. the 90th percentile  

f. the 99.9th percentile

g. the top 3 percent  

h. the bottom 1 percent
14) A lawnmower manufacturing company has the policy that it will repair free of charge any lawnmower manufactured by them if it breaks down within one year of purchase. Suppose the length of time a lawnmower works without needing repairs is normally distributed with mean time 4.8 years and standard deviation 1.8 years.

a. What percentage of lawnmowers will require the free repair? (break down within 1 year)  
b. What length of time does a lawnmower in the 99th percentile of time without needing repairs represent?

15) On a typical Saturday, Genuardis Markets reports that the mean amount of money spent by customers is $27.21 and a standard deviation of $7.93. The distribution is roughly normal.

a. The middle 95% of customers spent between what two amounts?  
b. If a person spends $10, in what percentile does that place her?  
c. If a person spends $40, she is in the top _____ percent of spending?  
d. What range of amounts do the middle 60% represent?  
e. If a customer spends in the top 2%, how much does he spend?  
f. If a customer spends in the 99.5th percentile, how much does he spend?  
g. What is the difference between the 99.9th percentile and the 99.99th percentile?

16) At a busy traffic intersection, the mean waiting time to go through the intersection is 3 minutes, 20 seconds with a standard deviation of 52 seconds. (Convert everything to seconds)

a. The middle 95% of drivers spent between what two amounts of waiting time (1 dec. place)?  
b. If a driver waits exactly two minutes, what percentile does that represent? (Assume the longer the wait, the higher the percentile)  
c. If a person waits 5 minutes he is in the top _____ percent of waiting?  
d. What percentage of drivers waits between 2 and 4 minutes?  
e. If a driver waits in the top 2%, how much time does he wait?  
f. How long will a driver have to wait to be in the bottom 0.1% of waiting times?
17) A special task force of a military unit requires that the recruits not be too tall or too short. Suppose 12% of the applicants are rejected because they are too tall and 18% because they are too short. If the height of an applicant is normally distributed with a mean of 69.4 inches and a standard deviation of 3.5 inches, determine the heights that define whether an applicant is accepted or rejected.

18) The height of an adult male is known to be normally distributed with a mean of 69 inches and a standard deviation of 2.5 inches. How high should a doorway be so that 99.5% of the adult males can pass through it without having to bend?

19) In order to qualify for a bowling tournament, bowlers bowl 20 games. The results are roughly normal. The average score was 167 with a standard deviation of 24.2. a) If the tournament is limited to the top 10% of the bowlers, what score did a bowler need to qualify for the tournament? b) If the bottom 15% will be able to participate in a special beginner’s tournament, what is the score which decides the bottom 15%?

20) In certain states, special education is decided by IQ scores. In a particular state, the mean IQ score is 101.5 with a standard deviation of 13.8. Any student in the top 5% or the bottom 8% is considered a special ed. student. Assuming that IQ’s are normal, find the IQ scores associated with those categories.

21) In a physical education class, a group of boys are asked to do sit-ups. Based on the results, the instructor will place them in 4 different classes - super, good, average, and challenged. The average number of sit-ups that the boys do is 36.5 with a standard deviation of 8.7. If the instructor wishes to place the boys into the classes based on the following criteria, what are the cutoff points for the number of sit-ups for each class (nearest whole number sit-up)?

   a. super - top 12%   b. good - next 28%   c. moderate - next 40%   d. challenged - bottom 20%